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Functional Quantification

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1 Introduction

Traditional formal semantics assumes that quantification in natural language operates only on atomic entities. However, much recent work in natural language semantics has shown the advantages of more complex forms of quantification that involve functions over atomic domains. Two areas of functional quantification have received special attention. One area concerns the so-called *functional* and *pair-list* interpretation of questions and copular sentences. Another area deals with the *wide scope* interpretation of indefinite NPs. Quantification over *Skolem functions* is used to model both kinds of phenomena. Some theories restrict the usage of Skolem functions to the simple case of *choice functions*: Skolem functions that map any non-empty set to an entity in this set.

Despite the similarity in the mechanisms that are used for describing functional readings and wide scope indefinites, no attempt has so far been made to bring them into one framework. This paper proposes such a unified mechanism. It accounts for the similar distribution of the two phenomena with different NPs and establishes formal relations between functional quantification and Generalized Quantifier Theory. These relations lead to a novel hypothesis about the class of quantifiers that license functional interpretations and about the reasons for their restricted distribution.

In a nutshell, the two general phenomena with which this article deals can be illustrated by the following two sentences.

(1) a. The (only) woman that every man loves is his mother.

b. Every man loves a (certain) woman.

Sentence (1a) illustrates a *functional reading*. In this reading, the pronoun *him* is "bound" by the noun phrase *every man* although it is not within its syntactic scope. Quite similarly, sentence (1b) has a reading where the indefinite *a* (*certain*) *woman* is interpreted as taking *wide scope* over the subject, although the subject is not within its syntactic scope.

Popular analyses of both phenomena involve Skolem functions under some version or another. To illustrate these approaches, consider first the following intuitive analyses of sentences (1a) and (1b).

(2) a. The (only) function in the set $\{f : f$ maps every man to a woman he loves$\}$ is the function that maps every man to his mother.

b. There is a choice function f such that every man loves $f(\{x : x$ is a woman$\})$.

We refer to nominal expressions such as *woman* and *woman that every man loves* as the *restricting predicate*, or *restriction*, of the relevant noun phrase. The functions in (2a) are in the denotation of the surface restriction *woman that every man loves*. Consequently, this analysis derives a "bound" reading of the pronoun *his* without assuming that *every man* takes the pronoun within its scope. Similarly, since the function *f* in (2b) applies to the restriction *woman* in its surface position, there is no standard scope of the object over the subject in this analysis.

However, there is one important difference between the two analyses. In the analysis (2a) of the functional reading, it is the restricting predicate itself (i.e. *woman that every man loves*) that is assumed to range over functions. In the functional analysis (2b) of the scope of indefinites, the function is used as a variable that only applies to the restricting predicate *woman*, but it does not belong to the restriction itself. Accordingly, existential quantification over functions in (2b) is assumed as a (possibly contextual) default mechanism, and not as the denotation of the indefinite article *a* (*certain*) in (1b). By contrast, the quantifier *the* (*only*) over functions in (2a) is assumed to be the denotation of the definite article *the* (*only*) in (1a).

The first aim of this paper is to bridge this gap between the two kinds of analyses. It will be argued that a unified analysis is not only justified by the technical and conceptual similarity between the two theories, but also because functional readings with copular sentences, questions and wide scope indefinites appear with the same class of NPs. The first step in unifying the two mechanisms, which is quite uncontroversial, is to generalize the choice function analysis of wide scope indefinites to Skolem functions of arbitrary arity. This is independently necessary for treating indefinites such as *a certain woman he knows*, where the pronoun *he* is bound from outside the indefinite. The second step we will take is to modify

Jacobson's (1994) analysis of functional readings in variable-free semantics. This modification will allow the Skolem function mechanism of wide scope indefinites to apply to the restrictive predicate also with functional copular sentences, while keeping the quantification over functions existential only.

In unifying Jacobson's account with the general treatment of indefinites, we will use an operator from objects of type $(ee)t$ (sets of functions from entities to entities) to binary $e(et)$ relations (functions from entities to sets of entities). Over finite domains this mapping can be one-to-one only under certain restrictions on its domain of $(ee)t$ objects. It is shown that this is naturally guaranteed when the quantifier within the functional NP (e.g. *every man* in (1a)) is what we call a *bounded* quantifier – an intersection of a principal filter and a principal ideal. This leads to the linguistically plausible hypothesis that only such quantifiers can give rise to functional readings.

The structure of this paper is as follows. Section 2 briefly overviews the problems of functional readings and wide scope indefinites, gives necessary technical details about previous approaches and discusses the motivation for a unified analysis. Section 3 introduces the mapping that allows an extended theory to treat both phenomena, and exemplifies its applications. Section 4 motivates the restrictions on the proposed mapping, and proves the relations between this restriction and the class of bounded quantifiers.

2 Functional readings and wide scope indefinites

2.1 Functional readings

The so-called *functional* reading of questions can be illustrated by the following familiar question-answer pairs.

(3) a. Which woman does every man love? His mother.
    b. Which woman does no man love? His mother-in-law.

The problem of interpreting questions that exhibit this kind of reading was discussed extensively in the literature.[1] It is often related (cf. Chierchia 1993) to the problem of *pair-list* readings of questions, as illustrated by the following short discourse.

[1] See Engdahl (1986:ch.4) and Groenendijk and Stokhof (1984:ch.3) for two classical works on this topic.

(4) Which woman does every man love? John loves Mary, Bill loves Sue, etc.

In order to analyze both kinds of readings that questions exhibit, it has been proposed that quantification over functions plays a role in the interpretative procedure. Similar mechanisms have been proposed for functional readings of *copular sentences* like the following.

(5) a. The woman that every man loves is his mother.

b. The woman that no man loves is his mother-in-law.

Sharvit (1999) convincingly argues that such copular sentences have the same distribution and syntactic/semantic properties of functional questions as in (3). In order to illustrate the mechanisms that will be considered, we will therefore concentrate on such indicative copular sentences, without getting into the more intricate semantics of questions.

A fully worked-out account of functional readings in copular sentences as in (5) is given in Jacobson (1994). Jacobson claims that sentences like (5a) and (5b) cannot be treated by giving the noun phrase *every man* sentential scope, as informally illustrated below.

(6) For every (no) man x, the woman that x loves is x's mother (mother-in-law).

One reason is syntactic: to obtain such an analysis of the sentences in (5), the noun phrases *every/no man* would have to cross a complex NP (*the woman that...loves*), which is generally acknowledged to be a scope island. For instance, sentence (7a) does not have the reading that is paraphrased in (7b).

(7) a. The woman that no man loves came to the party.

b. For no man x, the woman that x loves came to the party.

Moreover, noun phrases normally do not bind pronouns that are not within their syntactic scope (their *c-command domain*) as witnessed by the fact that the informal analysis in (8b) is not an available reading of sentence (8a).[2]

(8) a. The woman that no man loves pinched him.

[2]Sharvit (1999) claims, following Doron (1982), that if *no* in (8a) is replaced by *every*, then binding may sometimes become possible, but argues that this option should be derived by the same mechanism that derives pair-list readings with *every* as in (4). A similar point may be related to the fact that some speakers find the statement in (7b) more acceptable as a reading of (7a) when *no* is replaced by *every* or *each*. Since this paper does not deal with pair-list readings, these points are not directly relevant to its main purposes.

b. For no man x, the woman that x loves pinched x.

Another argument against a "scopal" analysis, which Jacobson attributes to Dahl (1981), is semantic. Consider sentence (9a) below. This sentence is clearly not equivalent with (9b), which is obtained by giving the noun phrase *no man* sentential scope.

(9) a. The only woman that no man loves is his mother-in-law.

b. For no man x, the only woman that x loves is his mother-in-law.

To see that, consider a situation in which John is a man who loves both his wife and his mother-in-law. In this situation (9a) is false. However, (9b) may still be true, as long as also other men do not love *only* their mother-in-law.

Jacobson's account of functional readings is based on her general theory of variable-free semantics for anaphora. The theory itself is introduced in much detail in Jacobson (1999,2000), and I will not try to review all of its parts here. Jacobson's assumptions that are important for our present purposes are the following, which for convenience are given the names (J1)-(J4).

(J1) An expression E that contains a "free pronoun" P denotes a function from entities of the standard type of P to entities of the standard type of E.

Consider for instance the noun phrase *the woman who gave him birth* or, equivalently *his mother* (as in (5)). Assume that the standard type of NPs is e and that this is also the standard type of the pronoun *him*. Jacobson therefore assumes that the whole NP denotes a function of type ee: a function from entities to entities. In the example, this is the function that maps every (male human) entity to its mother.

(J2) Transitive predicates like *love*, of the standard type $e(et)$, have a secondary meaning of type $(ee)(et)$ that ranges over ee functions in the object argument. This reading enables the subject NP to "bind" a pronoun within the object. The operator that derives this additional meaning of transitive predicates is denoted 'Z' and is defined as follows.

(10) $\mathrm{Z}_{(e(et))((ee)(et))} \stackrel{def}{=} \lambda R_{e(et)}.\lambda f_{ee}.\lambda x_e.R(f(x))(x)$

In words: the Z function maps a binary relation R to the relation $\mathrm{Z}(R)$ that holds exactly between those ee-type functions f and entities x that satisfy $R(f(x))(x)$. For instance, the following example (11a) is analyzed as in (11b).

(11) a. Every man loves his mother.

b. $\mathbf{every}'_{(et)((et)t)}(\mathbf{man}'_{et})(\mathrm{Z}(\mathbf{love}'_{e(et)})(\mathbf{his\_mother}'_{ee}))$
$\Leftrightarrow \mathbf{every}'(\mathbf{man}')(\{x : \mathbf{love}'(\mathbf{his\_mother}'(x))(x)\})$
$\Leftrightarrow \mathbf{man}' \subseteq \{x : \mathbf{love}'(\mathbf{his\_mother}'(x))(x)\}$

The determiner *every* here standardly denotes the subset relation between sets.

(J3) Items like the definite article *the*, the relative pronoun *that* and the copula *be* can range over ee functions as well as "ordinary" e-type entities. In essence, we can assume that such items denote the polymorphic *iota* operator, intersection operator and identity relation respectively.

(J4) Intransitive restricting predicates like *woman*, of the standard type et, have a second meaning of type $(ee)t$. This meaning ranges over ee functions and allows the restriction to combine with functional relative clauses. I use 'N' to denote the operator that derives this additional meaning of transitive predicates, and its meaning is defined as follows.

(12) $\mathrm{N}_{(et)((ee)t)} \stackrel{def}{=} \lambda P_{et}.\lambda f_{ee}.\forall x_e[P(f(x))]$

For instance, the $(ee)t$ denotation of the noun *woman* that the N operator derives is the set of ee functions that map each entity to a woman. Note that this set is empty when there are no women in the model.

For sake of exposition, I will use here a slightly modified version of the Z operator that Jacobson uses for binding. This revised operator, which is denoted 'Z^0', allows generalized quantifiers of type $(et)t$, rather than e type entities, to combine directly with the binary relation that is modified by the operator. Its definition follows.

(13) $\mathrm{Z}^0{}_{(e(et))(((et)t)((ee)t))} \stackrel{def}{=} \lambda R_{e(et)}.\lambda Q_{(et)t}.\lambda f_{ee}.Q(\lambda x_e.R(f(x))(x))$

This operator has essentially the same consequences of Jacobson's Z operator, but its arguments are now a quantifier and an ee function (in this order), instead of an ee function and an entity as in Jacobson's analysis. This revised formulation of Z only comes to allow a generalized quantifier such as *every man* in the relative clause *that every man loves* to "saturate" the subject argument of the transitive predicate *loves*, without getting into complex questions concerning the derivation of this option within a general categorial theory.[3]

Now we can get back to the sentences in (5) and illustrate their analysis in Jacobson's approach. A derivation of the meaning of sentence (5a) is summarized in figure 1. The type variable τ stands for any monomorphic type.

$$\dfrac{\dfrac{\text{the}}{\iota_{(\tau t)\tau}} \quad \dfrac{\dfrac{\text{woman}}{\mathrm{N}(\mathbf{woman'})} \quad \dfrac{\text{that}}{\cap_{(\tau t)((\tau t)(\tau t))}} \quad \dfrac{\dfrac{\text{every man}}{\mathbf{every'}(\mathbf{man'})} \quad \dfrac{\text{loves}}{\mathrm{Z}^0(\mathbf{love'})}}{\mathrm{Z}^0(\mathbf{love'})(\mathbf{every'}(\mathbf{man'}))}}{\mathrm{N}(\mathbf{woman'}) \cap \mathrm{Z}^0(\mathbf{love'})(\mathbf{every'}(\mathbf{man'}))}}{\iota(\mathrm{N}(\mathbf{woman'}) \cap \mathrm{Z}^0(\mathbf{love'})(\mathbf{every'}(\mathbf{man'}))) } \quad \dfrac{\dfrac{\text{is}}{id_{\tau(\tau t)}} \quad \dfrac{\text{his mother}}{\mathbf{his\_mother'}_{ee}}}{\lambda g_{ee}.g = \mathbf{his\_mother'}}$$

$$\iota(\mathrm{N}(\mathbf{woman'}) \cap \mathrm{Z}^0(\mathbf{love'})(\mathbf{every'}(\mathbf{man'}))) = \mathbf{his\_mother'}$$

Figure 1: Jacobson's derivation of meaning for sentence (5a)

It is not hard to verify that the result of the above derivation is tantamount to the following statement:

(14) There is only one ee function f s.t. f maps every man to a woman he loves, and this function is the function that maps every man to his mother.

This paraphrase captures the intuitive meaning of sentence (5a). Furthermore, this meaning is derived without any scope shifting of the noun phrase *every man*, which, as mentioned above, would have been problematic.

Alexopoulou and Heycock (2001) observe that copular sentences exhibit functional readings also in cases where their subject is not necessarily a singular definite as in (5). Some examples for this observation follow.

(15) a. One/a (certain) woman that every man loves is his mother.

b. One/a (certain) woman that no man loves is his mother-in-law.

(16) A woman that every/no man would be happy to see again is his childhood sweetheart.

(17) a. (The) two women that every Frenchman admires are his mother and Brigitte Bardot. (after Engdahl (1986:ch.4))

[3] Jacobson's mechanism achieves this using (a modified version of) the "Geach Rule" and Function Composition.

b. (The) two women that no Frenchman admires are his mother-in-law and Margaret Thatcher.

Jacobson does not mention such sentences, and their functional readings are not immediately captured by her mechanism. However, along the lines of Jacobson's proposal, it is natural to assume that pre-nominal items such as *one, a* and (*the*) *two* should also be analyzed as polymorphic operators, similar to the polymorphic *iota* operator that Jacobson assigns to the definite article. For instance, the items *one* and *a* in (15) and (16) could be analyzed as polymorphic existential determiners, of type $(\tau t)((\tau t)t)$. When τ is the ee type, of functions from entities to entities, this would allow the determiner to compose with a restricting predicate of type $(ee)t$ as in Jacobson's polymorphic analysis of the definite article.

However, such an analysis would be unnecessarily strong, because other NPs do not lend themselves so easily to functional readings. Consider for instance the following unacceptable sentences.

(18) ??At most/at least one woman that every man loves is his mother.

(19) ??No woman that every man would be happy to see again is his childhood sweetheart.

(20) ??Between two and three women that every Frenchman admires are his mother, Brigitte Bardot and possibly Isabelle Adjani.

In (20), for instance, it could be expected that the sentence would entail that there are between two and three functions that send each Frenchman to a woman he admires, in a similar way to the entailment from (17a) to the existence of two such functions. However, sentence (20) is quite incoherent. Thus, it would not be too helpful to assign a polymorphic meaning to all the determiners in natural language, since we only need a proper subset of them (if any) to range over functions. In this paper I propose that in fact, *no* determiner should range over functions. According to the proposed mechanism, functional quantification is only existential, and it is derived by the same general functional process that is responsible for the interpretation of wide scope indefinites. The unified process that will emerge will also cast some doubts on the usefulness of the other polymorphic entries that are assumed in (J3) and of the N operator that is assumed in (J4).[4]

[4]See the derivation in (39) below.

2.2 Wide scope indefinites

The main reason for introducing functions in the semantic analysis of indefinites is their ability to take scope beyond syntactic islands, which has received much attention in the literature since Fodor and Sag (1982). Consider for instance the following example.

(21) If some/a (certain) girl I know arrives to the party then John will be happy.

This sentence has the reading that is paraphrased in (22) below, which makes a statement about a "specific girl".

(22) There is a girl x I know such that if x arrives to the party then John will be happy.

We say that under this reading, the indefinite takes *wide scope* (WS) over the conditional. This reading is sometimes sloppily referred to as the *WS reading* of the indefinite. This behavior of indefinites is in sharp contrast with the behavior of other NPs. For instance, the following sentence in (23a) does not have the analysis that is paraphrased in (23b).

(23) a. If every girl arrives to the party then John will be happy.

b. For every girl x, if x arrives to the party then John will be happy.

This claim can be attested by considering that sentence (23a) can be intuitively false in situations where sentence (23b) is true. For instance, John may be happy if *any* girl comes to the party, while not wanting *all* the girls to come to the party together.

The special scopal behavior of indefinites calls for an explanation. In recent years, many authors have followed Reinhart (1992,1997) and Kratzer (1998), and assumed that the problem should be solved by allowing indefinites to be interpreted using *choice functions* (CFs), whose definition is given below.

Definition 1 (choice functions) *Let E be a non-empty set. A function f from $\wp(E)$ to E is a* choice function *over E iff for every $A \subseteq E$: if A is not empty then $f(A) \in A$.*

In extensional type logical frameworks like the one assumed throughout this paper, the set CH of *choice functions over type* τ is defined as follows:

(24) $CH^{\tau} \stackrel{def}{=} \lambda f_{(\tau t)\tau}.\forall P_{\tau t} \neq \emptyset[P(f(P))]$

Convention: we often write 'CH' instead of 'CH^e'.

The WS behavior of indefinites is treated using CFs by letting a free CF variable apply to the restriction of the indefinite. We assume that an existential quantifier applies to this variable at the matrix level, and derives the following interpretation of sentence (21).

(25) $\exists f[CH(f) \wedge [\mathbf{arrive}'(f(\mathbf{girl}')) \rightarrow \mathbf{glad}'(\mathbf{j}')]]$

The introduction of an existential quantifier over CFs – especially at levels lower than the matrix level – is controversial. Some authors, notably Kratzer (1998), favor a usage of CFs as "deictic" entities, without existential quantification over them. This controversy is not central for the purposes of this paper, and readers may easily modify the analyses below to the "deictic" version of CFs. Another problematic point which is not directly relevant to our purposes here is the treatment of indefinites with an empty restricting predicate. For example, consider (21) when there happens to be no girl in the given situation. In Winter (1997) and Winter (2001:ch.3) this case is given a solution using CFs of a higher type. For the sake of presentation I will not employ here this more complex analysis.

We have seen that indefinites with *some* and *a certain* show WS behavior beyond the "adjunct island" of the conditional. The same holds for such indefinites in other syntactic environments that behave like scope islands.[5] However, many other NPs (e.g. universal NPs as in (23a)) do not show this island-free behavior. It is therefore important to characterize those NPs to which the CF mechanism applies. This question is taken up in Winter (2001:ch.4), where it is argued that in addition to simple indefinites as in (21), the CF mechanism also applies to simple numeral indefinites (e.g. *three students*), *WH* phrases and singular and plural definite NPs. The WS potential of simple numerals can be easily illustrated by the interpretation (26b) of sentence (26a).

(26) a. If three girls I know arrive to the party then John will be happy.

b. There is a set A of three girls that I know such that if the girls in A arrive to the party then John will be happy.

The WS potential of *WH* elements is discussed (among others) by Reinhart (1997), using question-answer pairs like the following:

(27) Who will be offended if we invite which philosopher? John will be offended if we invite Putnam.

[5] See Ruys (1992) for a concise description of the facts.

To interpret the noun phrase *which philosopher*, early semantic theories of questions would have to assign it sentential scope over the conditional. Reinhart however shows that the CF mechanism can treat such effects of *WH in situ* in a similar fashion to the treatment of indicative indefinite that was reviewed above. As for definite NPs, because of their uniqueness requirement it is not easy to test their scopal behavior. I refer the reader to Winter (2001:ch.4) for other interpretative effects with definites that are accounted for by their CF interpretation.

Crucially, we have seen above that all these kinds of NPs – simple singular/plural (in)definites and WH elements – also lead to functional readings (cf. (3), (5) and (15)-(17)). Conversely, NPs as in (18)-(20) do not give rise to wide scope readings beyond islands, and therefore they do not require the CF analysis. Consider for instance the contrast between *at least one* in the following sentence and *some* in (21) above.

(28) If at least one girl (I know) arrives to the party then John will be happy.

This sentence does not have the reading that is paraphrased below.

(29) There is at least one girl I know x such that if x arrives to the party then John will be happy.

Similar unavailability of WS interpretation can also be illustrated for the other NPs in (18)-(20). This distinction between NPs where the CF analysis is available and other NPs where it is unavailable, is addressed by the syntactic-semantic mechanism that is proposed in Winter (2000, 2001:ch.4). In these works I propose that the two kinds of nominals correspond to two difference syntactic classes of DP structures. It is proposed that so-called *flexible DPs*, like simple (in)definites and WH phrases, are basically predicative, and therefore in argument position they are interpreted using CFs. Other DPs, which are called *rigid*, include complex numerals (e.g. *at least one* or *exactly one*) and universal quantifiers, and involve a full DP structure. Consequently, these DPs are purely quantificational and have no CF interpretation.

Of course, the parallelism we observed between WS interpretation and the availability of functional readings is not accounted for in the absence of a unified theory of both phenomena. However, before moving on to the details of this theory, we need to discuss a crucial ingredient of the proposal: general Skolem functions of arbitrary arity.

2.3 General Skolem functions

It is interesting to note that, quite independently of the scope of indefinites that have occupied semantic theories in recent years, some earlier works had proposed to use general *Skolem functions* (SFs) for capturing other semantic effects with indefinites and interrogatives. Intuitively, we can think of an n-ary Skolem function as a choice function with n parameters. While a choice function f maps any non-empty set X to a member of X, a general Skolem function f_n of arity n maps such a set X, together with tuple of n parameters $\langle x_1, ..., x_n \rangle$, to a member of X. Since we want to treat parameters in such tuples as "free variables" in Jacobson's variable-free framework, we assume that the input to a Skolem function is an n-place *function* that has the power-set $\wp(E)$ as its range. A variable-free Skolem function modifies such a function to a function that has E as its range. Formally:

Definition 2 (Skolem functions of arity $\geq$ 1) *Let E and A be non-empty sets. A function f from $(\wp(E))^A$ to E^A is a* Skolem function *iff for every function $g \in (\wp(E))^A$, for every $x \in A$: $(f(g))(x) \in g(x)$. If A is a Cartesian product of arity $n \geq 1$ then we say that f is a Skolem function of arity n.*

In a purely functional type-theoretical format, the set A cannot be a Cartesian product. However, Currying allows us to simulate products using functions and generalize the notion of Skolem functions. SFs of arity $n \geq 0$ are defined as objects of the following type scheme:

$$(\tau_1(...(\tau_n(\tau t))...))(\tau_1(...(\tau_n\tau)...)).$$

In this type scheme, the argument of an SF is a function of type $\tau_1(...(\tau_n(\tau t))...)$, which is isomorphic to a function from the Cartesian product $D_{\tau_1} \times ... \times D_{\tau_n}$ to the sets of τ-type elements. An SF maps such a function to a function of type $\tau_1(...(\tau_n\tau)...)$, which is isomorphic to a function from the Cartesian product $D_{\tau_1} \times ... \times D_{\tau_n}$ to τ-type elements.

The set SK^n of general Skolem functions of this type is defined as follows.

(30) $SK^n \stackrel{def}{=} \lambda f.\forall g_{\tau_1(...(\tau_n(\tau t))...)}\forall x_1...\forall x_n$
$[g(x_1)...(x_n) \neq \emptyset \rightarrow (g(x_1)...(x_n))((f(g))(x_1)...(x_n))]$

Convention: We usually assume that $\tau_1 = ... = \tau_n = \tau$, and say that a function in $SK^{\tau,n}$ is a *Skolem function* (of arity n) *over type* τ. We often write 'SK^n' instead of '$SK^{n,e}$'.

		g	h_1	h_2	h_3	h_4
a	$\mapsto$	$\{c\}$	c	c	c	c
b	$\mapsto$	$\{a,b\}$	a	a	b	b
c	$\mapsto$	$\{a,b\}$	a	b	a	b

Table 1: an $e(et)$ function and some ee functions

With this notation it is clear that the SF of arity 0 over type τ are the CFs over τ.

Consider the following example for a Skolem function f of arity 1. Assume that $E = \{a, b, c\}$. Let g be a function of type $e(et)$ that maps a to the singleton set $\{c\}$ and maps both b and c to the set $\{a, b\}$. Then the function $f(g)$ must be one of the four ee functions $h_1, ..., h_4$ that are described in table 1. Note that h_2 and h_3 illustrate the liberty of $f(g)$ to "choose" a different element of the set $\{a, b\}$ for each of the two "parameter values" b and c. It is in this sense that SFs of arbitrary arity are more powerful than CFs.

The close relationships between WS indefinites and general SFs on the one hand, and Jacobson's treatment of functional readings on the other hand, are most easily demonstrated by the following examples due to Groenendijk and Stokhof (1984:ch.3) and Hintikka (1986) respectively:

(31) Every man loves a (certain) woman – his mother.

(32) According to Freud, every man unconsciously wants to marry a certain woman – his mother.

Both Groenendijk & Stokhof and Hintikka propose to treat such examples using an existential quantifier over SFs that takes matrix scope. In the current setting, this means that the restricting predicate that the noun *woman* denotes must be a two-place relation, of type $e(et)$. This is surprisingly reminiscent of Jacobson's assumption (J4), according to which restricting predicates are systematically of a higher type. Jacobson's N operator maps the set of women to the set of ee functions that map any entity to some woman or other. Instead, now we need to map the set of women to a "parameterized set of women": an $e(et)$ function that maps any entity to *the set of women*. In view of this similarity, let us refer to the general operator by "N^0", which is formally defined as follows.

(33) $\mathrm{N}^0{}_{(et)(e(et))} \stackrel{def}{=} \lambda P_{et}.\lambda x_e.\lambda y_e.P(y)$

Using SFs, we can model discourses such as (31) and (32) by existential quantification over an SF at the matrix level. Even without getting into the technical derivation of the discourse binding in these examples, it is clear that existential quantification over SFs accounts for the interpretation of such sentences, which is intuitively similar to the interpretation of copular constructions such as (5). To give an illustration of this treatment, consider the following analysis of (31).

(34) $\exists f_{(e(et))(ee)}[SK^1(f) \wedge \mathbf{every'}(\mathbf{man'})(\mathrm{Z}(\mathbf{love'})(f(\mathrm{N}^0(\mathbf{woman'}))))]$

In words: there is an Skolem function f of arity 1, which maps the function $\mathrm{N}^0(\mathbf{woman'})$ – the constant function that maps every entity to the set of women – to an ee function h. This h function furthermore has the property that every man is in $\mathrm{Z}(\mathbf{love'})(h)$. More simply: every man loves the woman that h assigns to him. Assuming that the set of women is not empty, this is equivalent to the standard analysis of sentences like (31), with narrow scope existential quantification over e-type entities:

(35) $\forall x \in \mathbf{man'} \exists y \in \mathbf{woman'}[\mathbf{love'}(y)(x)]$

We see that in these examples, general SFs are employed to account for *narrow scope* readings of indefinites that due to the anaphora have a wide scope "functional flavor". It is curious to note that Reinhart's usage of the simpler 0-arity SFs (CFs) for deriving "ordinary" *wide scope* readings of indefinites was discovered rather late in the development of the theory.

In addition to the use of general Skolem functions in the treatment of indefinites as in (31) and (32), there are (at least) four other types of motivations that were given in the literature for their introduction:

1. Kratzer (1998) objects to the existential closure of CFs in Reinhart's analysis. Instead, Kratzer proposes to use general SFs as a means for deriving readings that under Reinhart's account require existential quantification over CFs with scope that is narrower than matrix scope.

2. Schlenker (1998) and Winter (1998,2001:ch.3) argue (independently) that certain sentences with indefinites show readings that are inexpressible using CFs alone and require SFs of arbitrary arity.[6]

[6]Following Hintikka, Schlenker also uses general SFs for deriving "branching readings" with indefinites. I will not discuss this phenomenon here.

3. In Winter (2001:ch.3) it is argued that SFs are required in order to eliminate problematic analyses that are derived using CFs for indefinites with free pronouns (e.g. *a woman he knows*).

4. Chierchia (2001) argues that certain effects with indefinites, including weak crossover and *de re* interpretations, require introduction of SFs of arbitrary arity.

I will not discuss these issues in this paper, but simply assume, as in most works on this subject, that general SFs are required for the treatment of indefinites. The exact restrictions on their introduction is a topic that is still under extensive investigation.[7] The next section shows how this assumption allows us to achieve a unified treatment of functional readings and WS indefinites.

3 A unified mechanism for functional readings and wide scope indefinites

In the previous section we have seen technical, intuitive and distributional reasons to assume that "functional readings" and "wide scope indefinites" are two phenomena that should be derived by the same mechanism. Technically, the SF treatment of the scope of indefinites involves functions from entities to entities, like the ones that were independently employed by Jacobson and others for treating sentences with functional readings. Intuitively, the "wide scope functional" interpretation of discourses as in (31) and (32) seems to be essentially the same kind of thing as the functional interpretation of questions such as (3) or copular sentences such as (5). From a distributional point of view, we saw that the same NPs that are treated using SFs and can show WS effects beyond islands also show "functional" effects. It is therefore natural to expect the two phenomena to be amenable to the same treatment.

However, as mentioned in the introduction, one link is missing between the two kinds of theories that were reviewed above. Jacobson assumes that the restricting predicate in NPs with functional readings *ranges* over functions. Consequently, in her account any NP should in principle allow quantification over functions. By contrast, the CF treatment assumes that choice functions *apply to* the

[7] In addition to Chierchia (2001), see also Schlenker (1998) and Schwartz (2001) for some recent work in progress that deal with this topic.

restricting predicate, which denotes a set of "ordinary" entities of type e. Quantification over CFs – if needed at all, which Kratzer doubts – is only existential, and applies syncategorematically – independently of the syntax/semantics of the NP. The same holds of the way SFs of higher arity are used for treating scopal effects with indefinites.

The main argument of this section is that this discrepancy can be resolved by renouncing Jacobson's general polymorphic scheme of quantification over functions. In the proposed modification of her mechanism, functions are involved in the internal semantics of the NP as part of the variable-free mechanism in assumptions (J1) and (J2). However, the restricting predicate and the determiner range over ordinary entities and are standardly treated using the general SF mechanism. This allows us to renounce Jacobson's assumption (J4) about the N operator. Further polymorphism as in (J3) will be needed only with the copula construction and not with determiners or relative pronouns.

To achieve this unified analysis, the main new part of the proposal is an operator that maps one-place predicates over functions, of type $(ee)t$, to binary predicates of type $e(et)$. The operator, which is called 'RG' (for "range"), is defined as follows.

(36) $\mathrm{RG} \stackrel{def}{=} \lambda F_{(ee)t}.\lambda x_e.\lambda y_e.\exists f \in F[f(x) = y]$

Intuitively, if F is a set of ee functions, then $\mathrm{RG}(F)$ maps each entity x to the set of entities y that elements of F assign to it.[8] For instance, in table 1 above, $\mathrm{RG}(\{h_1, h_2, h_3, h_4\}) = g$.

To illustrate how this operator allows us to unify the two mechanisms, consider again Jacobson's analysis of sentence (5a), which was given in figure 1 above. In the revised analysis, the denotation of the "gapped" constituent *every man loves* still denotes the same $(ee)t$ set of functions as in Jacobson's analysis. This is the set $F = \mathrm{Z}^0(\mathbf{love'})(\mathbf{every'}(\mathbf{man'}))$ – the set of functions that map every man to something he loves. The RG operator maps F to a binary predicate – the function that maps every man to the *set of* things he loves, provided that every man loves something.[9] Let us henceforth denote this binary relation $\mathrm{RG}(F)$ by 'S'. The binary relation S should be intersected with the denotation of common noun

[8]Alternatively, we can standardly view an ee function f as the binary relation $R_f \stackrel{def}{=} \lambda x.\lambda y.y = f(x)$. Then $\mathrm{RG}(F)$ is simply the union of the relations that correspond to functions in F: $\mathrm{RG}(F) = \cup_{f \in F} R_f$. I thank Nissim Francez for pointing this out to me.

[9]If there is a man who does not love anything, then F is empty and consequently $\mathrm{RG}(F)$ sends everything to the empty set.

woman. This can be achieved by lifting the set $\mathbf{woman'}$ using the N^0 operator that was defined in (33) above, and was used in the treatment of sentence (31) in (34).[10] Consequently, the restricting predicate *woman that every man loves* denotes the function that maps every man to the set of *women* he loves. Let us refer to this binary relation $\mathrm{N}^0(\mathbf{woman'}) \cap S$ by the letter 'R'. To the binary relation R we can apply the general SF mechanism, as in (34). It is easier to see how this SF mechanism works with indefinites. Therefore, let us first consider how the meaning of sentence (37) below is derived. This sentence (=(15a)) is a slight variation on Jacobson's example (5a).

(37) A (certain) woman that every man loves is his mother.

The analysis of this example using SFs is given in figure 2, where EC stands for "existential closure" – here of an SF of arity 1. In this analysis, the Skolem

$$\frac{\text{every man}}{\mathbf{every'}(\mathbf{man'})} \quad \frac{\text{loves}}{\mathrm{Z}^0(\mathbf{love'})}$$

$$\frac{\text{woman}}{\mathrm{N}^0(\mathbf{woman'})} \quad \frac{\text{that}}{\cap_{(\tau t)((\tau t)(\tau t))}} \quad \frac{\mathbf{every'}(\mathbf{man'}) \quad \mathrm{Z}^0(\mathbf{love'})}{\mathrm{RG}(\mathrm{Z}^0(\mathbf{love'})(\mathbf{every'}(\mathbf{man'})))}$$

$$\frac{\text{a}}{f_{(e(et))(ee)}} \quad \frac{\mathrm{N}^0(\mathbf{woman'}) \quad \cap_{(\tau t)((\tau t)(\tau t))} \quad \mathrm{RG}(\mathrm{Z}^0(\mathbf{love'})(\mathbf{every'}(\mathbf{man'})))}{\mathrm{N}^0(\mathbf{woman'}) \cap \mathrm{RG}(\mathrm{Z}^0(\mathbf{love'})(\mathbf{every'}(\mathbf{man'})))} \quad \frac{\text{is}}{id_{\tau(\tau t)}} \quad \frac{\text{his mother}}{\mathbf{his\_mother'}_{ee}}$$

$$\frac{f_{(e(et))(ee)} \quad \mathrm{N}^0(\mathbf{woman'}) \cap \mathrm{RG}(\mathrm{Z}^0(\mathbf{love'})(\mathbf{every'}(\mathbf{man'})))}{f(\mathrm{N}^0(\mathbf{woman'}) \cap \mathrm{RG}(\mathrm{Z}^0(\mathbf{love'})(\mathbf{every'}(\mathbf{man'}))))} \quad \frac{id_{\tau(\tau t)} \quad \mathbf{his\_mother'}_{ee}}{\lambda g_{ee}.g = \mathbf{his\_mother'}}$$

$$\frac{f(\mathrm{N}^0(\mathbf{woman'}) \cap \mathrm{RG}(\mathrm{Z}^0(\mathbf{love'})(\mathbf{every'}(\mathbf{man'})))) \quad \lambda g_{ee}.g = \mathbf{his\_mother'}}{f(\mathrm{N}^0(\mathbf{woman'}) \cap \mathrm{RG}(\mathrm{Z}^0(\mathbf{love'})(\mathbf{every'}(\mathbf{man'})))) = \mathbf{his\_mother'}}$$

$$\frac{f(\mathrm{N}^0(\mathbf{woman'}) \cap \mathrm{RG}(\mathrm{Z}^0(\mathbf{love'})(\mathbf{every'}(\mathbf{man'})))) = \mathbf{his\_mother'}}{\exists f_{(e(et))(ee)}[SK^1(f) \wedge f(\mathrm{N}^0(\mathbf{woman'}) \cap \mathrm{RG}(\mathrm{Z}^0(\mathbf{love'})(\mathbf{every'}(\mathbf{man'})))) = \mathbf{his\_mother'}]}\ EC$$

Figure 2: Derivation of meaning for sentence (37)

function f applies to the denotation of the restricting predicate *woman that every man loves*: the binary relation R that sends each man to the set of women he loves. Assuming that $R(x)$ is non-empty for every man x, any SF sends R to an ee function that maps every man to one of the women he loves. The statement that is derived in figure 2 claims that one of these ee functions is the mother function, which is the desired interpretation of sentence (37).

The proposed treatment of sentence (37) as described above is presented in a way that highlights the main differences from Jacobson's treatment. These differences (cf. figure 1 vs. figure 2) are the introduction of the RG operator and the application of SFs to the restricting predicate. However, a more general treatment

[10] Later we will see that this special operator is actually not needed here, and its postulation reflects a more general property of the "binding" mechanism.

is obtained if we adopt the mechanism of Winter (2001:ch.3) for handling "free variables". This mechanism uses a special functional type constructor "$\rightarrow$" in order to distinguish the type of denotations that contain "free variables" from the type of other functions.[11] An expression of type $\tau \rightarrow \sigma$ denotes a function from D_τ to D_σ, but it behaves compositionally like an expression of type σ that contains a "gap" (free variable) of type τ. For instance, a transitive predicate such as *love* gets the standard type $e(et)$. By contrast, a binary relation like the relation R that is denoted by the expression *woman that every man loves* gets the type $e \rightarrow (et)$: it behaves compositionally like an ordinary noun of type et, but involves a"free variable" of type e. In order to capture this double nature of expressions with gapped meanings, we use the following categorial rule.

$$\frac{\Gamma, \sigma_1 \vdash \sigma_2}{\Gamma, \tau \rightarrow \sigma_1 \vdash \tau \rightarrow \sigma_2}\ \mathrm{ACOND} \qquad \frac{X, y_1(x_\tau) \Rightarrow y_2(x_\tau)}{X, y_1 \Rightarrow y_2}$$

This rule, presented here in sequent format with the appropriate semantics, is called ACOND (for "Argument Conditionalization"), and it is a restricted version of the general Conditionalization rule of the Lambek Calculus (see Van Benthem, 1991). An example for its application is the following derivation of meaning for the constituent *that every man loves* in (37). Unlike Jacobson, we assume now that the relative pronoun *that* standardly denotes the *monomorphic* intersection function of type $(et)((et)(et))$. The derivation of meaning for the constituent *every man loves* proceeds as in figure 2. Recall that we denote this meaning by 'S'. Accordingly, we assume the following types and meanings:

(38) $[\![\text{that}]\!] = \cap_{(et)((et)(et))} \stackrel{def}{=} \lambda A_{et}.\lambda B_{et}.\lambda x_e.A(x) \wedge B(x)$

$[\![\text{every man loves}]\!] = S_{e\rightarrow(et)} \stackrel{def}{=} \mathrm{RG}(\mathrm{Z}^0(\mathbf{love'})(\mathbf{every'}(\mathbf{man'})))$

Composition of these two types and meanings is achieved by the following application of ACOND:

$$\frac{(et)((et)(et)),\ et \vdash (et)(et)}{(et)((et)(et)),\ e \rightarrow (et) \vdash e \rightarrow ((et)(et))}\ \mathrm{ACOND}$$

To see the use semantics of this rule, note the following valid derivation (using

[11] Jacobson (1999) identifies expressions that contain "free variables" by their syntactic category rather than by their semantic type, and changes the categorial syntax accordingly.

application):

$\cap,\ S(x) \Rightarrow \cap(S(x))$,

which is tantamount to:

$\cap,\ (\lambda y.S(y))(x) \Rightarrow (\lambda y.\cap(S(y)))(x)$

Using ACOND, we can use this derivation as follows:

$$\frac{\cap,\ (\lambda y.S(y))(x) \Rightarrow (\lambda y.\cap(S(y)))(x)}{\cap,\ S \Rightarrow \lambda y.\cap(S(y))}\ \mathrm{ACOND}$$

Intuitively, when composing $\cap$ and S, this rule amounts to: introduction of a "fresh free variable" y, application of $\cap$ to $S(y)$ and abstraction over y. Motivations for this particular formulation and for the "$\rightarrow$" constructor are given in Winter (2001:ch.3), and I will not repeat them here.

In a similar way, the ACOND rule allows us to derive the meaning of the constituent *woman he loves*, without applying the N^0 operator to *woman* as in figure 2. This derivation is given below.

(39)
$$\frac{\dfrac{(et)(et),\ et \vdash et}{et,\ e \rightarrow ((et)(et)) \vdash e \rightarrow (et)}\ \mathrm{ACOND} \qquad \mathbf{woman'},\ \cap(S(x)) \Rightarrow \mathbf{woman'} \cap (S(x))}{\dfrac{\mathbf{woman'},\ (\lambda y.\cap(S(y)))(x) \Rightarrow (\lambda y.\mathbf{woman'} \cap (S(y)))(x)}{\mathbf{woman'},\ \lambda y.\cap(S(y)) \Rightarrow \lambda y.\mathbf{woman'} \cap (S(y))}\ \mathrm{ACOND}}$$

Note that the result is the same as the relation R, which was obtained above by intersection of S with $\mathrm{N}^0(\mathbf{woman'})$ (cf. figure 2). Formally, let $\cap_1$ and $\cap_2$ denote the intersection operators of one-place and two-place predicates respectively. Then we have:

(40)
$$\begin{aligned}
&[\![\text{woman that every man loves}]\!] = R \\
&= (\mathrm{N}^0(\mathbf{woman'})) \cap_2 S \\
&= (\lambda y.\lambda z.\mathbf{woman'}(z)) \cap_2 S \\
&= \lambda y.((\lambda z.\mathbf{woman'}(z)) \cap_1 S(y)) \\
&= \lambda y.\mathbf{woman'} \cap_1 S(y)
\end{aligned}$$

In words, for any binary relation S: the intersection of S with the relation that sends each entity to the set of women, is the relation that sends each entity y to the set of women in $S(y)$. This means that when the second conjunct in the construction *woman that...* ranges over functions as in Jacobson's account, the outcomes of the N^0 operator are now derived rather than stipulated.[12]

The revised analysis of sentence (37) using the ACOND rule is given in figure 3, with the abbreviations S and R for the denotations of the expressions *every man loves* and *woman that every man loves* respectively, as derived above. The meaning that is derived in this way is equivalent to the meaning that is derived using the more *ad hoc* mechanism in figure 2. It is important to mention that the introduction of the free SF variable in this analysis (and the following one) is only for the sake of presentation. In fact, the ACOND rule as introduced above is designed to get rid of such free variables, as explained in Winter (2001:ch.3).

With the ACOND rule, it is also no longer necessary to adopt Jacobson's assumption (J3) that items such as relative pronouns (e.g. *that*) or the definite article (i.e. *the*) are polymorphic. We adopt the treatment in Winter (2001:ch.3) of the definite article as a predicate modifier that imposes singularity on the restricting predicate:

(41) $[\![\text{the}]\!] = \mathbf{the}'_{(et)(et)} \stackrel{def}{=} \lambda A_{et}.\lambda x_e.|A| = 1 \wedge A(x)$

Using these assumptions, the analysis of sentence (5a) in figure 4 becomes analogous to the analysis of sentence (37) in figure 3.[13] A similar analysis becomes now possible for copular sentences with bare numerals as illustrated in (17). We adopt the common assumption that the numeral *two* is a predicate modifier, similarly to the definite article. Concretely, given a cardinality function *card* on "plural" e-type entities, we can assume the following denotation for the numeral *two*:

(42) $[\![\text{two}]\!] = \mathbf{two}'_{(et)(et)} \stackrel{def}{=} \lambda A_{et}.\lambda x_e.card(x) = 2 \wedge A(x)$

This makes the analysis of (17) similar to the analysis of (5a) in figure 4.[14]

[12]Note that for cases such as (31) or (32) above, where there is no functional conjunct, an operation like N^0 is still required in order to introduce the "implicit variable" within the noun *woman*. I do not address here the linguistic status of such implicit variables.

[13]The only material difference between the two analyses is that the SF variable corresponds to the indefinite article in figure 3, and to a null element in figure 4. In fact, in Winter (2001) it is argued that SF variables correspond to null elements in both cases, and the deviation from this treatment is here only a matter of presentation.

[14]Post-copular nominals such as *his mother and Brigitte Bardot* are treated using the ACOND rule. Suppose (for the sake of presentation alone) that *and* denotes here the i-sum operator of Link (1983). This operator, of type $e(ee)$ composes using ACOND with the e-type denotation of *Brigitte Bardot* and the $(e \to e)$-type denotation of *his mother*, to derive another $(e \to e)$-type function that maps each man to the i-sum of his mother and Brigitte Bardot. A similar analysis can be derived within the more complex treatment of plurals and predication that is proposed in Winter (2001:ch.4).

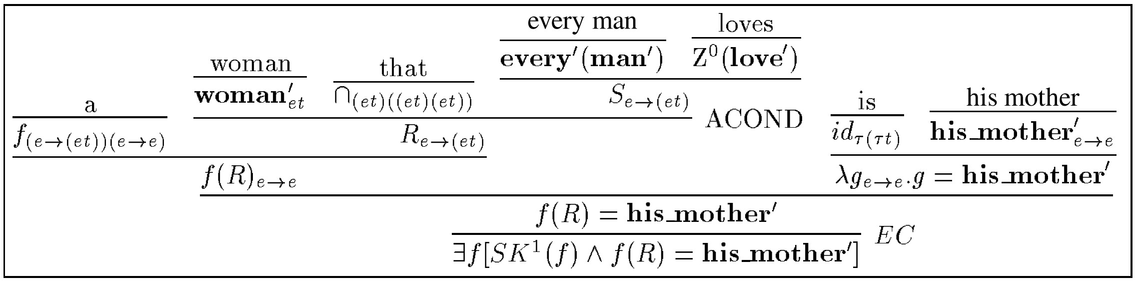

Figure 3: Revised derivation of meaning for sentence (37)

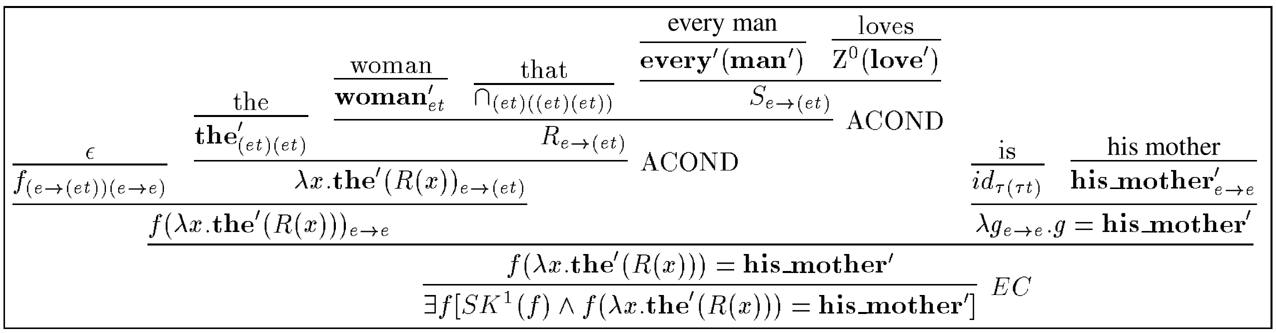

Figure 4: Derivation of meaning for sentence (5a)

Note that in the proposed modification of Jacobson's system, the only polymorphic item in the analysis of functional readings is the copula *be*. This is not coincidental. The polymorphism of *be* is what allows predicates such as *be his mother* to range over functions of type $e \to e$. With a monomorphic transitive predicate such as *love* or *pinch* this is not possible. In the proposed system, there are two ways to analyze a verb phrase such as *love his mother*. The object *his mother* is of type $e \to e$. The transitive predicate *love* is standardly of type $e(et)$. One way to compose their denotations is using ACOND; another is using the Z (or Z^0) function and direct application. These two options are illustrated below.

(43) a. $\mathbf{love}'_{e(et)},\ \mathbf{his\_mother}'_{e\to e} \overset{\text{ACOND}}{\Rightarrow} (\lambda x.\mathbf{love}'(\mathbf{his\_mother}'(x)))_{e\to(et)}$

b. $\mathrm{Z}(\mathbf{love}')_{(e\to e)(et)},\ \mathbf{his\_mother}'_{e\to e} \Rightarrow (\lambda x.\mathbf{love}'(\mathbf{his\_mother}'(x))(x))_{et}$

The first option accounts for the deictic interpretation of the pronoun (as in *Mary loves his mother*). The second option accounts for the bound interpretation of the pronoun (as in *every man loves his mother*). However, the verb phrase cannot apply directly to NPs that denote $e \to e$ functions, which accounts for the impossibility to get a functional reading in non-copular sentences such as the following (cf. Sharvit (1999)).

(44) a. The woman that no man admires hates his mother.

b. The woman that no man loves pinched him.(=(8a))

Assuming that no man admires his mother-in-law, sentence (44a) cannot mean something like "a mother-in-law always hates her son-in-law's mother". In other words: the sentences in (44), as opposed to (5b), are (correctly) not given a functional analysis. This is so because of the assumption that ordinary transitive predicates, as opposed to *be*, do not have a polymorphic meaning.[15]

Let us summarize the main proposals of this section:

- Sets of ee functions are mapped to binary relations using the RG operator.
- This allows the general SF mechanism to derive functional readings.
- The analysis of functional readings does not require Jacobson's N operator or polymorphic meanings of items such as *that* or *the*. These are treated using a more general variable-free compositional mechanism.
- Polymorphism of the copula *be* is required in order to account for its licensing of functional readings as opposed to other transitive predicates.

Note that SFs of arity greater than one are needed in order to analyze sentences with more than one pronoun in the post-copular NP. For example:

(45) The present that every man will send to every woman is the present that she asked him to send her.

For sake of exposition, I will not consider such complex examples in the sequel.

4 Generalized quantifiers and functional readings

Mapping sets of functions to binary relations using the RG operator may potentially result in loss of information. To see that, note that in a model with n entities,

[15]In fact, as argued in Winter (2001:ch.4), it may be advantageous to treat copulas as meaningless rather than polymorphic, but this is hardly relevant for our purposes here.

the cardinality of the $(ee)t$ domain is $2^{(n^n)}$, whereas the cardinality of the $e(et)$ domain is only $(2^n)^n = 2^{(n^2)}$. Consequently, the RG operator is not one-to-one. This means that the present proposal is, at least in principle, less expressible than Jacobson's original quantification over functions. In this section it is argued that this loss of expressibility is not only innocuous, but actually desired. In the mechanism that was introduced in the previous section, a restricting predicate such as *woman that Q loves* denotes a set of functions F for any quantifier Q. This section introduces a notion of *closed sets of functions*, for which the RG operator is one-to-one, and shows that the set F is closed if and only if Q is a *bounded quantifier*: an intersection of a principal filter and a principal ideal. We will then hypothesize that it is exactly the bounded quantifiers that are licensed in functional interpretations of questions and copular sentences.

To illustrate the intuitive reasoning that underlies the formal discussion in this section, consider the following contrast.

(46) A woman that $\left\{ \begin{array}{l} \text{no} \\ \text{*at most one} \end{array} \right\}$ man loves is his mother-in-law.

In Jacobson's analysis, the denotation of the constituent *woman that no man loves* is the set of functions that send each man to a woman he does not love. Let us denote this set of functions by F_{no}. Assuming that F_{no} is not empty, the relation $\mathrm{RG}(F_{no})$ sends every man to the set of women he does not love. Applying any Skolem function of arity 1 to $\mathrm{RG}(F_{no})$ gives us again a function in F_{no}. This is not the case when we consider the determiner *at most one*, which does not support a functional reading in (46). Let us denote the set of functions that send at most one man to a woman he loves by $F_{\leq 1}$. Assume that the men are John and Bill and that the women are Mary and Sue. Assume further that John loves Mary and hates Sue, and that Bill loves Sue and hates Mary. The set $F_{\leq 1}$ includes, among other functions, also the following two functions f_1 and f_2:

$$\begin{array}{lll} f_1: & \mathrm{John} \mapsto \mathrm{Mary}, & \mathrm{Bill} \mapsto \mathrm{Mary} \\ f_2: & \mathrm{John} \mapsto \mathrm{Sue}, & \mathrm{Bill} \mapsto \mathrm{Sue} \end{array}$$

This means that $\mathrm{RG}(F_{\leq 1})$ sends both John and Bill to the set $\{\mathrm{Mary}, \mathrm{Sue}\}$. Consequently, a Skolem function may map $\mathrm{RG}(F_{\leq 1})$ to the function that sends each of the two men to the woman he loves. But this function is not in $F_{\leq 1}$. This formal distinction between *no woman* and *at most one woman* will be given a general characterization in this section, in terms of bounded and unbounded quantifiers. It will be hypothesized that (un)boundedness of quantifiers is responsible for contrasts as in (46).

4.1 Closed sets of functions and bounded quantifiers

Consider first the following definition.

Definition 3 (closed sets of functions) *Let $F \subseteq B^A$ be a set of functions from A to B. The* closure of F *is the set of functions $\overline{F} \subseteq B^A$ that is defined by:*

$\overline{F} \stackrel{def}{=} \{f \in B^A$ *: for every $x \in A$ there is $g \in F$ s.t. $f(x) = g(x)$*$\}$.

We call F a closed set of functions *if $F = \overline{F}$.*

Example: Consider the functions in table 1 above. The set $\{h_1, h_2\}$ is closed. By contrast, the set $\{h_1, h_4\}$ is not closed: its closure is $\{h_1, h_2, h_3, h_4\}$.
Note that $F \subseteq \overline{F}$ trivially holds for any set of functions F. It is also easy to observe the following fact.

Fact 1 *If F and G are sets of ee functions then $\mathrm{RG}(F) = \mathrm{RG}(G)$ iff $\overline{F} = \overline{G}$.*

As a result, the RG operator is one-to-one when it is restricted to apply to closed sets of functions:

Corollary 2 *If F and G are closed sets of ee functions then $\mathrm{RG}(F) = \mathrm{RG}(G)$ iff $F = G$.*

A related fact about closed sets of functions, which was informally mentioned and illustrated above, concerns their relations with Skolem functions and the RG operator.

Fact 3 *A non-empty set F of ee functions is closed iff for every Skolem function f of arity 1 over type e: $f(\mathrm{RG}(F)) \in F$.*

This fact uses the RG operator to establish a relation between two classes of Skolem functions:

$SK^{1,e}$: the SFs of arity 1 over type e;
$SK^{0,ee}$: the SFs of arity 0 over type ee (CFs over ee).

It claims that (only) over *closed* sets of ee functions, the function composition $f \circ \mathrm{RG}$ is a function in $SK^{0,ee}$ for any function $f \in SK^{1,e}$.

Proof of fact 3
($\Rightarrow$): Assume that F is a non-empty closed set of functions and that f is an SF of arity 1 over type e. For every x: from $F \neq \emptyset$ it follows that $\mathrm{RG}(F)(x) \neq \emptyset$, and by definition of SFs: $f(\mathrm{RG}(F))(x) \in \mathrm{RG}(F)(x)$. By definition of RG, this

Noun Phrase:	Quantifier bounded by:	
every student	$\mathbf{student'}$	E
no student	$\emptyset$	$E \setminus \mathbf{student'}$
every student but no teacher	$\mathbf{student'}$	$E \setminus \mathbf{teacher'}$
every student but Mary	$\mathbf{student'} \setminus \{\mathbf{m'}\}$	$E \setminus \{\mathbf{m'}\}$
no student but Mary	$\{\mathbf{m'}\}$	$E \setminus (\mathbf{student'} \setminus \{\mathbf{m'}\})$

Table 2: Bounded quantifiers

means that for every x there is $g \in F$ such that $f(\mathrm{RG}(F))(x) = g(x)$. Hence $f(\mathrm{RG}(F)) \in \overline{F}$. But $F = \overline{F}$ by closure of F, hence $f(\mathrm{RG}(F)) \in F$.
($\Leftarrow$): Assume that for every Skolem function f of arity 1 over type e: $f(\mathrm{RG}(F)) \in F$. By definition of SFs, this means that for every function g_{ee}: if g sends every x to a member of $\mathrm{RG}(F)(x)$ then $g \in F$. But this means that $\overline{F} \subseteq F$, hence $\overline{F} = F$, so F is closed. □

Thus, we know that for a non-empty closed set of functions F, sequential application of RG and a Skolem function maps F to one of its members. Recall that the set F is generated from an restricting predicate such as (*woman that*) *every man loves*, which is derived by application of the Z^0 operator to a quantifier Q (e.g. *every man*) and a binary relation R (e.g. *love*). Let us use the following abbreviation:

(47) $F_{QR} \overset{def}{=} \mathrm{Z}^0(Q_{(et)t})(R_{e(et)}) = \lambda f_{ee}.Q(\lambda x_e.R(f(x))(x))$

Further modification of the set F_{QR} within the relative clause is immaterial for our present purposes, hence ignored. The main theorem of this section characterizes the class of quantifiers Q that guarantee that F_{QR} is a closed set of functions for every R. It will be shown that this set is precisely the set of *bounded* quantifiers: those quantifiers that are an intersection of a principal filter (e.g. the denotation of *every student*) and a principal ideal (e.g. the denotation of *no teacher*). This class of quantifiers is officially defined below.

Definition 4 (bounded quantifiers) *A quantifier $Q \subseteq \wp(E)$ is* bounded by the sets X and $Y \subseteq E$ *iff* $Q = \{A \subseteq E : X \subseteq A \subseteq Y\}$.

Table 2 gives some NPs that denote bounded quantifiers with the sets that bound them.

The main theorem of this section is stated below.

Theorem 4 *Let E be a denumerable set. For any quantifier $Q \subseteq \wp(E)$: the set of functions F_{QR} is closed for any binary relation $R \subseteq E \times E$ iff Q is bounded.*

The proof of this theorem makes use of the *convexity*[16] property of quantifiers, which is defined below.

Definition 5 (convex quantifiers) *A quantifier $Q \subseteq \wp(E)$ is* convex *iff for all $A \subseteq B \subseteq C \subseteq E$: if $A \in Q$ and $C \in Q$ then $B \in Q$.*

For instance, the noun phrase *between three and five students* denotes a convex quantifier in any model, but this is not true of the disjunctive noun phrase *exactly three or exactly five students*.

The following simple fact will be useful in proving theorem 4.

Fact 5 *A quantifier $Q \subseteq \wp(E)$ is bounded iff Q is convex and closed under arbitrary intersections and unions. In this case $Q = \{A \subseteq E : \cap Q \subseteq A \subseteq \cup Q\}$.*

The "only if" direction in the proof of theorem 4, which is its harder part, follows from fact 5 together with lemma 6 below.

Lemma 6 *Let E be a denumerable set. For any quantifier $Q \subseteq \wp(E)$: if the set of functions F_{QR} is closed for any binary relation $R \subseteq E \times E$, then Q is convex and closed under (denumerable) intersections and unions.*

Proof Assume that for a quantifier $Q \subseteq \wp(E)$, the set of functions F_{QR} is closed for any binary relation $R \subseteq E \times E$. We will show that Q is convex and closed under denumerable unions and intersections. The proofs of the convexity and closure properties of Q are all similar. We consider the *identity relation* R_0: the relation that satisfies $\mathrm{R}_0(x)(y)$ iff $x = y$. In each case we show that $F_{Q\mathrm{R}_0}$ is not closed if Q does not have the respective property. Thus, in each case the proof constructs a subset of $F_{Q\mathrm{R}_0}$ with a closure that is not contained in $F_{Q\mathrm{R}_0}$, and thereby contradicts the assumption that F_{QR} is closed for any binary relation R.

Convexity of Q: Suppose, for contradiction, that there are $A \subset B \subset C \subseteq E$ s.t. $A, C \in Q$ but $B \notin Q$. We will show that the set of functions $F_{Q\mathrm{R}_0}$ is not closed. Let us denote $b \in B \setminus A$ and $c \in C \setminus B$, for arbitrary b and c. These exist by our assumptions about A,B and C. Consider the following functions:

$$f_0(x) = \begin{cases} x & x \in A \\ c & x \in B \setminus A \\ b & x \in E \setminus B \end{cases} \qquad g_0(x) = \begin{cases} x & x \in B \\ b & x \in E \setminus B \end{cases} \qquad h_0(x) = \begin{cases} x & x \in C \\ b & x \in E \setminus C \end{cases}$$

[16] Thijsse (1983), who introduced this notion into generalized quantifier theory, gave it the misnomer *continuity*, which was used in some subsequent works.

Observe the following equalities:

$\{x \in E : \mathrm{R}_0(f_0(x))(x)\} = A$
$\{x \in E : \mathrm{R}_0(g_0(x))(x)\} = B$
$\{x \in E : \mathrm{R}_0(h_0(x))(x)\} = C$

By definition of $F_{Q\mathrm{R}_0}$ and the non-convexity assumption about Q it therefore follows that $f_0, h_0 \in F_{Q\mathrm{R}_0}$, but $g_0 \notin F_{Q\mathrm{R}_0}$.
However, by definition of f_0, g_0 and h_0, we have for every $x \in E$:

$$\begin{array}{lll} x \in B & \Rightarrow & g_0(x) = x = h_0(x) \\ x \in E \setminus B & \Rightarrow & g_0(x) = b = f_0(x) \end{array}$$

That is, for every $x \in E$ there is $f \in \{f_0, h_0\}$ s.t. $g_0(x) = f(x)$. Hence $g_0 \in \overline{\{f_0, h_0\}} \subseteq \overline{F_{Q\mathrm{R}_0}}$, which means that $\overline{F_{Q\mathrm{R}_0}} \neq F_{Q\mathrm{R}_0}$. By assuming non-convexity of Q, we contradicted the assumption that F_{QR} is a closed set of functions for any relation R. Hence: Q is convex.

Closure of Q under finite intersections: Suppose, for contradiction, that there are $A, B \subseteq E$ s.t. $A, B \in Q$ but $A \cap B \notin Q$. As in the proof above of the convexity of Q, we show that the set of functions $F_{Q\mathrm{R}_0}$ is not closed.
Let us denote $a \in A \setminus B$ and $b \in B \setminus A$, for arbitrary a and b. These exist by our assumptions about A and B.
Consider the following functions:

$$f_0(x) = \begin{cases} x & x \in A \\ a & x \in E \setminus A \end{cases} \qquad g_0(x) = \begin{cases} x & x \in B \\ b & x \in E \setminus B \end{cases}$$

$$h_0(x) = \begin{cases} a & x \in E \setminus (A \cup B) \\ b & x \in A \setminus B \\ a & x \in B \setminus A \\ x & x \in A \cap B \end{cases}$$

Observe the following equalities:

$\{x \in E : \mathrm{R}_0(f_0(x))(x)\} = A$
$\{x \in E : \mathrm{R}_0(g_0(x))(x)\} = B$
$\{x \in E : \mathrm{R}_0(h_0(x))(x)\} = A \cap B$

By definition of $F_{Q\mathrm{R}_0}$ and the assumption about Q it therefore follows that $f_0, g_0 \in F_{Q\mathrm{R}_0}$, but $h_0 \notin F_{Q\mathrm{R}_0}$.

However, by definition of f_0, g_0 and h_0, we have for every $x \in E$:

$$\begin{array}{lll} x \in E \setminus (A \cup B) & \Rightarrow & h_0(x) = a = f_0(x) \\ x \in B \setminus A & \Rightarrow & h_0(x) = a = f_0(x) \\ x \in A \setminus B & \Rightarrow & h_0(x) = b = g_0(x) \\ x \in A \cap B & \Rightarrow & h_0(x) = x = f_0(x) = g_0(x) \end{array} .$$

That is, for every $x \in E$ there is $f \in \{f_0, g_0\}$ s.t. $h_0(x) = f(x)$. Hence $h_0 \in \overline{\{f_0, g_0\}} \subseteq \overline{F_{Q\mathrm{R}_0}}$, which means that $\overline{F_{Q\mathrm{R}_0}} \neq F_{Q\mathrm{R}_0}$. This contradiction to our assumption about the closure of $F_{Q\mathrm{R}_0}$ entails that Q is closed under finite intersections.

Closure of Q under denumerable intersections: This proof is a generalization of the proof for the finite case. Suppose, for contradiction, that there is a denumerably infinite set $\mathcal{A} \subseteq Q$ s.t. $\cap\mathcal{A} \notin Q$.
We can assume without loss of generality that the members of $\mathcal{A}$ form a decreasing sequence $A_1 \supset A_2 \supset ... \supset A_i \supset ...$.[17]
Let us use the notations $a_1 \in A_1 \setminus A_2,\ a_2 \in A_2 \setminus A_3, ...,\ a_i \in A_i \setminus A_{i+1},\ ...$, for arbitrary $a_1, a_2, ..., a_i, ...$. These exist by our assumptions about $\mathcal{A}$.
Consider the following functions f_i, for $i \geq 0$, and h_0:

$$f_i(x) = \begin{cases} x & x \in A_{i+1} \\ a_{i+1} & x \in E \setminus A_{i+1} \end{cases} \qquad h_0(x) = \begin{cases} a_1 & x \in E \setminus A_1 \\ a_{j+1} & x \in A_j \setminus A_{j+1},\ \text{for } j \geq 1 \\ x & x \in \cap\mathcal{A} \end{cases}$$

Observe the following equalities:
$\{x \in E : \mathrm{R}_0(f_i(x))(x)\} = A_{i+1}$, for every $i \geq 0$
$\{x \in E : \mathrm{R}_0(h_0(x))(x)\} = \cap\mathcal{A}$
By definition of $F_{Q\mathrm{R}_0}$ and the assumption about Q it therefore follows that $f_i \in$

[17] Justification: let the sequence $A_1, A_2, ...A_i, ...$ be some ordering of the members of $\mathcal{A}$. Consider the indices i_k, s.t. $i_1 = 1$ and for every $k \geq 1$:

$$i_{k+1} \stackrel{def}{=} \begin{cases} l_0 & i_k \neq 0;\ \text{there is } l > i_k \text{ s.t. } \cap_{i=1}^{i_k} A_i \supset \cap_{i=1}^{l} A_i; \\ & \text{and } l_0 \text{ is the minimal } l \text{ with this property} \\ 0 & \text{otherwise} \end{cases}$$

Suppose that there is no k for which i_k is 0. Then the infinite decreasing sequence $B_j \stackrel{def}{=} \cap_{k=1}^{j} A_{i_k}$ satisfies $B_j \in Q$ for every j (by closure of Q under finite intersections), and $\cap_{j=1}^{\infty} B_j = \cap\mathcal{A}$. Hence, $B_j|_{j=1}^{\infty}$ is the required sequence.
Otherwise, there is a finite k_0 s.t. $i_{k_0} = 0$, and consequently there is a finite subset $\mathcal{A}' \subset \mathcal{A}$ s.t. $\cap\mathcal{A}' = \cap\mathcal{A}$. But then $\cap\mathcal{A}' = \cap\mathcal{A} \notin Q$ is in contradiction to the closure of Q under finite intersections.

F_{QR_0} for every $i \geq 0$, but $h_0 \notin F_{\mathrm{QR}_0}$.
However, by definition of f_i and h_0, we have for every $x \in E$:

$$\begin{array}{lll} x \in E \setminus A_1 & \Rightarrow & h_0(x) = a_1 = f_0(x) \\ x \in A_1 \setminus A_2 & \Rightarrow & h_0(x) = a_2 = f_1(x) \\ \ldots & & \\ x \in A_i \setminus A_{i+1} & \Rightarrow & h_0(x) = a_{i+1} = f_i(x) \\ \ldots & & \\ x \in \cap\mathcal{A} & \Rightarrow & h_0(x) = x = f_0(x) = f_1(x) = \ldots = f_i(x) = \ldots \end{array}$$

That is, for every $x \in E$ there is $f \in \{f_i\}_{i=0}^{\infty}$ s.t. $h_0(x) = f(x)$. Hence $h_0 \in \overline{\{f_i\}_{i=0}^{\infty}} \subseteq \overline{F_{\mathrm{QR}_0}}$, which means that $\overline{F_{\mathrm{QR}_0}} \neq F_{\mathrm{QR}_0}$. This contradiction to our assumption about the closure of F_{QR_0} entails that Q is closed under infinitely denumerable intersections.

Closure of Q under denumerable unions: The proof is using similar constructions to the ones in the proofs that Q is closed under denumerable intersections.
For the finite case, suppose, for contradiction, that there are $A, B \subseteq E$ s.t. $A, B \in Q$ but $A \cup B \notin Q$.
We again denote $a \in A \setminus B$ and $b \in B \setminus A$, for arbitrary a and b. These exist by our assumptions about A and B.
We consider the following functions:

$$f_0(x) = \begin{cases} x & x \in A \\ a & x \in E \setminus A \end{cases} \qquad g_0(x) = \begin{cases} x & x \in B \\ b & x \in E \setminus B \end{cases}$$

$$h_0(x) = \begin{cases} a & x \in E \setminus (A \cup B) \\ x & x \in A \cup B \end{cases}$$

And the rest of the proof for finite unions is similar to the proof for finite intersections.
For the infinite case, suppose for contradiction that there is a denumerably infinite set $\mathcal{A} \subseteq Q$ s.t. $\cup\mathcal{A} \notin Q$. Again, without loss of generality we can assume that the members of $\mathcal{A}$ form an increasing sequence $A_1 \subset A_2 \subset \ldots \subset A_i \subset \ldots$.
We use the notations $a_1 \in A_2 \setminus A_1,\ a_2 \in A_3 \setminus A_2, \ldots,\ a_i \in A_{i+1} \setminus A_i,\ \ldots$, for arbitrary $a_1, a_2, \ldots, a_i, \ldots$. These exist by our assumptions about $\mathcal{A}$.
We consider the following functions f_i, for $i \geq 1$, and h_0:

$$f_i(x) = \begin{cases} x & x \in A_i \\ a_{j+1} & x \in A_{j+1} \setminus A_j, \text{ for } j \geq i \\ a_1 & x \in E \setminus \cup\mathcal{A} \end{cases} \qquad h_0(x) = \begin{cases} a_1 & x \in E \setminus \cup\mathcal{A} \\ x & x \in \cup\mathcal{A} \end{cases}$$

And the rest of the proof for denumerably infinite unions is similar to the proof for denumerably infinite intersections. □

We can now complete the proof of theorem 4.

Proof of theorem 4

(⇒): Follows from fact 5 and lemma 6.

(⇐): For the "⇐" direction we assume that $Q \subseteq \wp(E)$ is bounded by $X, Y \subseteq E$ and show that F_{QR} is a closed set of functions for any $R \subseteq E \times E$. It is enough to show that any $f_0 \in \overline{F_{QR}}$ is also in F_{QR}. Thus, we need to show that the set $A_0 \stackrel{def}{=} \{x \in E : R(f_0(x))(x)\}$ is in Q, which holds iff $X \subseteq A_0 \subseteq Y$.

1. To show that $A_0 \subseteq Y$, we assume $x_0 \in A_0$ and show $x_0 \in Y$.
   From $f_0 \in \overline{F_{QR}}$ it follows that some $g_0 \in F_{QR}$ satisfies $g_0(x_0) = f_0(x_0)$.
   Because $g_0 \in F_{QR}$, we conclude from the definitions of F_{QR} and Q that $\{x \in E : R(g_0(x))(x)\} \subseteq Y$. (i)
   From the assumption $x_0 \in A_0$, it follows that the relation $R(f_0(x_0))(x_0)$ holds. Because $g_0(x_0) = f_0(x_0)$, we conclude that $R(g_0(x_0))(x_0)$ holds too. (ii)
   Facts (i) and (ii) entail that x_0 is in Y.

2. To show that $X \subseteq A_0$, we assume $x_0 \in X$ and show $x_0 \in A_0$. The proof is symmetrical to the proof above that $A_0 \subseteq Y$:
   Because $f_0 \in \overline{F_{QR}}$, it follows that some $g_0 \in F_{QR}$ satisfies $g_0(x_0) = f_0(x_0)$.
   Because $g_0 \in F_{QR}$, we conclude from the definitions of F_{QR} and Q that $X \subseteq \{x \in E : R(g_0(x))(x)\}$. From $x_0 \in X$ it follows that $R(g_0(x_0))(x_0)$ holds.
   Because $g_0(x_0) = f_0(x_0)$, we conclude that $R(f_0(x_0))(x_0)$ holds too, which means that $x_0 \in A_0$.

□

4.2 The distribution of quantifiers in functional readings

Consider the following examples.

(48) The/A woman that $\left\{\begin{array}{l}\text{every/no man (but John)}\\ \text{*at most one man}\\ \text{?exactly/at least one man}\end{array}\right\}$ loves is his mother.

(49) Which woman does $\left\{\begin{array}{l}\text{every/no man (but John)}\\ \text{*at most one man}\\ \text{?exactly/at least one man}\end{array}\right\}$ love? His mother.

These examples show a contrast in acceptability of functional readings between bounded quantifiers such as *every, no* etc. and unbounded quantifiers such as *exactly one, at most/least one* etc. The following hypothesis makes a natural connection between this linguistic contrast and the formal properties that were proven in the previous subsection.

(50) **Hypothesis**: *the* RG *operator applies only to closed sets of functions.*

Let us examine how this hypothesis makes the desired connection. As shown by theorem 4, it is exactly the bounded quantifiers that generate closed sets of functions. Thus, if hypothesis (50) is correct, then the functional mechanism is analyzed as sensitive to the (un)boundedness property of the quantifier. This is due to theorem 4. Such sensitivity is not at all trivial to explain in compositional frameworks, because the functional mechanism applies at a much higher syntactic level than the level where the quantifier is present. However, theorem 4 makes the necessary connection between the restrictions on the set of functions that is generated, and the generalized quantifier that participates in its generation.

But *why* should the RG operator be sensitive to whether the set of functions it applies to is closed or not? Fact 3 gives at least a partial answer to this question. According to this fact, is exactly the closed sets that guarantee that the Skolem functions of arity 1 over entities work as Choice functions over ee functions. In other words: a Skolem function, when it applies to a denotation like $\mathrm{RG}(\llbracket$*woman that every man loves*$\rrbracket)$, does not return "unnatural functions" that were not already in the basic denotation of *woman that every man loves*. Thus, if the hypothesis in (50) is correct, then in a sense it makes the grammar of functional quantification semantically "optimal": functional readings derive "natural functions" exactly in the cases when they are grammatically licensed.[18]

Despite this attractiveness of hypothesis (50), there are (at least) two potential empirical problems that it has to face. One problem concerns the felicitous functional interpretation of sentences such as the following, which is based on examples due to Alexopoulou and Heycock (2001).

(51) The woman that almost every/no man loves is his mother.

[18] I believe that a similar reasoning underlies Barwise and Cooper's (1981) semantic account of the grammaticality of *there* sentences with various NPs, using a non-triviality assumption.

The natural interpretation of the noun phrase *almost every man* is the following.

(52) $\{A \subseteq E : 1 \leq |\overline{A} \cap \mathbf{man}'| \leq n\}$,
where n is a (small) number that is determined by the context.

This quantifier in not bounded. However, sentence (51) can also be interpreted as in the rough paraphrase below.

(53) There is a (small) set of men B s.t. the woman that every/no man except for the men in B loves is his mother.

In other words: the "exception set" B that *almost* quantifiers require can take here sentential scope, which allows the noun phrase *almost every/no man* to be interpreted as a bounded quantifier.[19]

Another problem for the hypothesis in (50) may come from certain claims that are made by Sharvit (1999:(18)-(21)). Sharvit considers plural sentences in Hebrew that are parallel to the following English sentence.[20]

(54) The woman that most of the students invited was their mother.

However, with noun phrases such as *most of the students*, also discourse anaphora as in the following example may appear and complicate the picture.

(55) Most of the students admire their mother. They invited her.

By contrast, discourse anaphora is unacceptable with the other quantifiers that illustrated (un)availability of functional readings in the above examples. For instance:

(56) No/at most one student hates his mother. *He invited her.

This means that the contrast we observed above between *no* and *at most one* cannot originate from different discourse anaphora potentials, and therefore it supports hypothesis (50). The functional reading in Sharvit's example (54) may result from discourse anaphora and therefore it does not seriously challenge this hypothesis. For recent works that deal with discourse anaphora using a mechanism of functional quantification see Steedman (1999) and Peregrin and Von Heusinger (2001).

[19]This is not always the case with *almost* quantifiers. For instance, a sentence such as *every Frenchman admires almost every actress*, is compatible with a situation where one Frenchman admires every actress but Brigitte Bardot, while another Frenchman admires every actress but Isabelle Adjani.

[20]Sharvit also judges as felicitous sentences like (54) with *more than two* and *at most two* instead of *most of the*. However, my Hebrew informants disagree, and consider these cases considerably worse than the Hebrew parallel of (54).

5 Conclusions

In this paper we have seen intuitive, distributional and technical support for the claim that "functional" readings of questions and copular sentences and "wide scope" readings of indefinites are two names for the same phenomenon. One simple operator that maps sets of functions to binary relations was used as the key for unifying separate mechanisms that had been previously introduced for treating these phenomena using Skolem functions. This unified mechanism establishes some new relations between functional quantification and generalized quantifier theory. In particular, it was shown that there are both empirical and mathematical reasons to expect functional quantification to be restricted to the newly introduced class of bounded quantifiers. Of course, many hard linguistic-logical questions about functional quantification in natural language are still open. However, I believe that the results in this paper exemplify the benefits that can be gained by the on-going study into this challenging area.

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