Nominal quantification, composition and evaluation

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1 Introduction

The denotations of natural language determiners like `every`, `all`, `most`, `at least two`, `exactly five` are traditionally analysed as being relations between sets of individuals (or equivalently functions from sets into functions from sets into truth-values).

Examples:
$[\![\texttt{most}]\!] = \lambda P.\lambda Q.[|P \cap Q|/|P| \geq 0.8]$
$[\![\texttt{exactly five}]\!] = \lambda P.\lambda Q.[|P \cap Q| = 5]$

One of the properties that determiners share is that they restrict quantification to the set provided by the restrictor, so that the set $Q - P$ is irrelevant.

Examples:

(1) a. Most children sleep.
    b. Most children are children who sleep.

(2) a. Exactly five children sleep.
    b. Exactly five children are children who sleep.

Quantification is restricted to the set of children in the sense that the set of sleepers which are not children is completely irrelevant to the truth or falsity of the sentences (1a) and (2a).

Put differently, there seem to be no natural language determiners which denote e.g.:

$[\![\texttt{?}]\!] = \lambda P.\lambda Q.[|Q - P| = 5]$

Brief mention of `only` – not a determiner, therefore not a counterexample:

(3) Only children sleep.

$[\![\texttt{only}]\!] = \lambda P.\lambda Q.[Q \subseteq P]$?

We can formulate this property by saying that all determiners have the following property:

For all M and all $A, B \subseteq M$:
$Q_M(A)(B) = Q_M(A, A \cap B)$

If determiners denote relations between sets of individuals, then it is somewhat surprising why determiners should be conservative.

How can we explain this fact?

2 The basic idea

Peters and Westerståhl (2006, p. 140): "one function of [expressions that signify relations between sets of individuals] is to restrict the domain of quantification to the denotation of the corresponding noun, i.e. the restriction argument".

But: why should these expressions have this function?

Remark: if the denotation of the determiner phrases did not make reference to the scope set, we would not have to explain why determiner phrases denote only certain such functions but not others. Put differently, it is because we analyse determiner phrases as denoting functions from scope sets to truth-values that we have to explain why only certain such functions are possible denotations.

Alternative: Instead of saying that (i) the denotation of determiner phrases makes reference to a scope set, and that (ii) one function of determiner phrases is to restrict the domain of quantification to the denotation of the corresponding noun, I propose that the denotation of natural language determiners does not make reference to a scope argument.

The basic idea is that the meaning of determiner phrases is to introduce a set variable (standing for a subset of the restrictor set) and impose a certain restriction on it.

In DRT some NPs (e.g. quantifying NPs) are analysed as denoting quantifiers, while some other (e.g. personal pronouns) are analysed as denoting variables plus a restriction on them.

So instead of unifying NP denotations by claiming that all NPs denote generalised quantifiers (as done for example in van der Does (1996)), I will explore the other direction – that of unifying the analysis of NP denotations by claiming that NPs essentially denote variables plus restriction.

3 First step – `at least four men`

Determiner denotation:
$[\![\texttt{at least four}]\!] = \lambda Y.\langle X; X \subseteq Y \wedge |X| \geq 4\rangle$

Combination of determiner and noun denotation:
$[\![\texttt{at least four}]\!]([\![\texttt{men}]\!]) = \lambda Y.\langle X; X \subseteq Y \wedge |X| \geq 4\rangle(MEN) =$

Determiner phrase denotation:
$[\![\texttt{at least four men}]\!] = \langle X; X \subseteq MEN \wedge |X| \geq 4\rangle$

Composition with unary predicate:
$\oplus_A(\langle X; X \subseteq MEN \wedge |X| = 4\rangle, \langle SING(\_\__A); \emptyset\rangle) =$

$\langle SING(X); \{X; X \subseteq MEN \wedge |X| \geq 4\}\rangle$

Subject and object NP denotations are combined with the predicate by means of different operators. $\oplus_A$ assigns the 'highest' semantic role to the NP denotation, whereas $\oplus_P$ assigns the second-highest semantic role to the NP denotation.

The merge operations must distinguish the two placeholders. Note that this is also required if the merge operation is functional application, since in order saturate a function in a particular order, it is necessary to 'know' which variable to saturate 'next'.

Desideratum: $\langle X; \text{Condition}(X)\rangle$ should be the same as $\langle Y; \text{Condition}(Y)\rangle$.

In order to make the merge operations independent of the particular variable name, the merge operations alter the variable names by attaching 1 to the subscript.

$\oplus_S(\langle X; X \subseteq MEN \wedge |X| = 4\rangle, \langle SING(\_\_S); \epsilon\rangle) = \langle SING(X_1); \{X_1; X_1 \subseteq MEN \wedge |X_1| \geq 4\}\rangle$

Evaluation – first version:
$\langle SING(X_1); \{X_1; X_1 \subseteq MEN \wedge |X_1| \geq 4\}\rangle$ is true iff
$\exists X_1.[X_1 \subseteq MEN \wedge |X_1| \geq 4 \wedge X_1 \subseteq SING]$

Problem: this kind of evaluation is not general enough – if we replace the quantifier `at least four` with `exactly four`, we see why.

$[\![$exactly four men sing.$]\!] = \langle SING(X_1); \{X_1; X_1 \subseteq MEN \wedge |X_1| = 4\}\rangle$ is true iff
$\exists X_1.[X_1 \subseteq MEN \wedge |X_1| = 4 \wedge X_1 \subseteq SING]$

i.e. there is a set X_1, whose individuals are both men and singing, and whose cardinality is exactly four. If this were the meaning of the sentence `Exactly four men sing.` then the sentence would be true if the set of **all** men who sing was e.g. 17, since in this case it is true that there is a set whose cardinality is 4 and whose individuals are both men and singing.

4 Second step – `exactly four men`

What this illustrates is that we need to make sure that the set X_1 is the **intersection** of the set of men and the set of singers. Essentially this is the price for not refering to the scope set in the denotation of the determiner phrase.

To rectify this, the evaluation operation needs to be able to 'see' the restrictor set. To do this, I assume that the denotation of determiner phrases is not of the form $\langle X; COND(X)\rangle$ but $\langle X; REST; COND(X)\rangle$. So the denotation of the two determiner phrases is:

$[\![$`at least four men`$]\!] = \langle X; MEN; X \subseteq MEN \wedge |X| \geq 4\rangle$

$[\![$`exactly four men`$]\!] = \langle X; MEN; X \subseteq MEN \wedge |X| = 4\rangle$

Evaluation – second version:

$[\![$`exactly four men sing.`$]\!] = \langle SING(X_1); \{X_1; MEN; X_1 \subseteq MEN \wedge |X_1| = 4\}\rangle$ is true iff

$\exists X_1.[X_1 \subseteq MEN \wedge |X_1| = 4 \wedge X_1 = MEN \cap SING]$

5 Third step – composition with binary predicate

Next, I will illustrate the semantic composition and evaluation of:

(4) `Four professors examined ten students.`

Combination of $[\![$`ten students`$]\!]$ with $[\![$`examined`$]\!]$:

$\oplus_P(\langle X; ST; X \subseteq ST \wedge |X| = 10\rangle, \langle EXAMINE(\_\_{}_A, \_\_{}_P); \emptyset\rangle) =$
$\langle EXAMINE(\_\_{}_A, X_1); \{\langle X_1; ST; X_1 \subseteq ST \wedge |X_1| = 10\rangle\}\rangle$

Combination of $[\![$`examined ten students`$]\!]$ with $[\![$`four professors`$]\!]$:

$\oplus_A(\langle X; PROF; X \subseteq PROF \wedge |X| = 4\rangle, \langle EXAMINE(\_\_{}_A, X_1); \{\langle X_1; ST; X_1 \subseteq ST \wedge |X_1| = 10\rangle\}\rangle) =$
$\langle EXAMINE(X_1, X_{11}); \{\langle X_{11}; ST; X_{11} \subseteq ST \wedge |X_{11}| = 10\rangle, \langle X_1; PROF; X_1 \subseteq PROF \wedge |X_1| = 4\rangle\}\rangle$

Evaluation – first possibility:

$\langle EXAMINE(X, Y); \{\langle Y; ST; Y \subseteq ST \wedge |Y| = 10\rangle, \langle X; PROF; X \subseteq PROF \wedge |X| = 4\rangle\}\rangle$
iff
$\exists Y.[Y \subseteq ST \wedge |Y| = 10 \wedge Y = ST \cap \{y | \langle EXAMINE(X, y); \{\langle X; PROF; X \subseteq PROF \wedge |X| = 4\rangle\}]$
iff
$\exists Y.[Y \subseteq ST \wedge |Y| = 10 \wedge Y = ST \cap \{y | \exists X.[X \subseteq PROF \wedge |X| = 4 \wedge X = PROF \cap \{x | EXAMINE(x, y)\}]\}]$

Evaluation – second possibility:

$\langle EXAMINE(X, Y); \{\langle Y; ST; Y \subseteq ST \wedge |Y| = 10\rangle, \langle X; PROF; X \subseteq PROF \wedge |X| = 4\rangle\}\rangle$
iff
$\exists X.[X \subseteq PROF \wedge |X| = 4 \wedge X = PROF \cap \{x | \langle EXAMINE(x, Y); \{\langle Y; ST; Y \subseteq ST \wedge |Y| = 10\rangle\}]$
iff
$\exists X.[X \subseteq PROF \wedge |X| = 4 \wedge X = PROF \cap \{x | \exists Y.[Y \subseteq ST \wedge |Y| = 10 \wedge Y = ST \cap \{y | EXAMINE(x, y)\}]\}]$

Evaluation schema:

$\langle R_i(\ldots, X, \ldots); Q \cup \{\langle X; REST; COND(X)\rangle\}\rangle$
iff
$\exists X.[COND(X) \wedge X = REST \cap \{x | \langle R_i(\ldots, x, \ldots); Q\rangle\}]$,

where Q is a possibly empty set of quantification statements.

The evaluation does not impose an order in which the relation has to be evaluated.

As a result of two different ways of evaluating a relation between two sets of individuals, the sentence is true in two different types of situations.

So far, only distributive interpretations of sentences can be computed. Next, I turn to the analysis of the collective readings.

6 Fourth step – collective interpretations

In order to analyse collective interpretations, I follow Verkuyl and van der Does (1995) and:

- introduce a new domain of entities and a new interpretation function (i.e. a new model) according to which the denotation of an individual constant is a singleton set $\{a\}$, the denotation of collections of individuals is $\{a, b, c\}$, and the extension of a predicate is not a set of individuals anymore, but a subset of the powerset of individuals, e.g. $\{\{a, b\}, \{c\}\}$.
- introduce partitions

The basic idea is to represent the reading according to which three men arrived individually by $\{\{a\}, \{b\}, \{c\}\} \in [\![\texttt{arrive}]\!]$ and the reading according to which three men arrived as a group as $\{\{a, b, c\}\} \in [\![\texttt{arrive}]\!]$.

So if the denotation of `arrive` is e.g. $\{\{a\}, \{b\}, \{c\}, \{d, e\}\}$, where $\{a\}, \{b\}, \{c\}$ denote men, and $\{d, e\}$ denotes a collection consisting of two women, then it is true that three men arrived individually, but false that three men arrived as a group.

Illustrations of sets which partition $Y = \{a, b, c\}$ in the intended sense:

- $X_1 = \{\{a\}, \{b\}, \{c\}\}$
- $X_2 = \{\{a\}, \{b, c\}\}$
- $X_2 = \{\{b\}, \{a, c\}\}$
- $X_2 = \{\{c\}, \{a, b\}\}$
- $X_2 = \{\{a, b, c\}\}$

Illustrations of sets which don't partition $Y = \{a, b, c\}$ in the intended sense:

- $X_1 = \{\{a\}, \{b\}\}$
- $X_2 = \{\{a, b\}, \{b, c\}\}$
- $X_2 = \{\{b, e\}, \{a, c\}\}$

A set X partitions a set Y (in symbols PART(X,Y)) iff (i) X is a subset of the powerset of Y, the union of all elements of X equals Y, and all elements of X are pairwise disjoint.

Evaluation schema – with partitions:

$\langle R_i(\ldots, X, \ldots); Q \cup \{\langle X; REST; COND(X)\rangle\}\rangle$
iff
$\exists X.[COND(X) \wedge \exists P.[PART(P, X) \wedge P = REST \cap \{x | \langle R(\ldots, x, \ldots); Q\rangle\}]]$

Example:

$[\![\texttt{three men arrived}]\!]$
iff
$\exists X.[X \subseteq \{x | \{x\} \in [\![\texttt{men}]\!]\}, |X| = 3 \ \wedge$
$\exists P.[PART(P, X) \wedge P = [\![\texttt{men}]\!] \cap \{x | \langle [\![\texttt{arrive}]\!](x); Q\rangle\}]]$

This sentence is therefore true if there is a partition P of the set X which covers the whole intersection between the denotation of `men` and the denotation of `arrive`. Crucially, no restriction is imposed on partition itself. Therefore the sentence may be true if:

- $P = \{\{a\}, \{b\}, \{c\}\}$
- $P = \{\{a\}, \{b, c\}\}$
- $P = \{\{b\}, \{a, c\}\}$
- $P = \{\{c\}, \{a, b\}\}$
- $P = \{\{a, b, c\}\}$

where $\{a\}, \{b\}, \{c\} \in [\![men]\!]$

7 Conclusion

- The denotation of a determiner phrase is not a function from scope sets to truth values.
- Instead, it is a triple consisting of a set variable, the restrictor, and a condition on these two entities.
- Subject and object NP denotations are combined with the verb by means of different semantic operations. These operations rename variables, so that it does not matter whether the the meaning of an NP is analysed as $X; REST; COND(X)$ or $Y; REST; COND(Y)$.
- There is more than one semantic operation – if we assume that different semantic operations are symbolised by different syntactic operations, then we come close to explaining why in so many languages the core arguments are realised in different ways.
- Relations between two or more sets of individuals can be evaluated in different ways. The evaluation depends among other things on the context of interpretation. This is how different scope ambiguities involving quantified NP are analysed.

- The difference between distributive, collective and cummlative ‘readings’ is attributed to different possible partitions. As in Verkuyl and van der Does (1995) the sentence `Three men arrived` is not ambiguous – it leaves the exact partition underspecified.
- Non-conservative determiners cannot be expressed, given this analysis of quantified NP denotations, composition and evaluation.
- The analysis of NP denotations can be unified as (i) introducing a variable and (ii) specifying a restriction on it.

References

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