Against generalized quantifiers (and what to replace them with)

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SFB 732 Stuttgart, 06.11.08

Generalized quantifier theory – the basic idea

- The sentence

  `Exactly three students smoke.`

  is true if and only if

  $|STUDENT \cap SMOKE| = 3$

- `Exactly three students` denotes that function $\mathbf{F}_1$, which when applied to a set B results in the truth value 1 if and only if:

  $[\![\texttt{exactly three students}]\!](B) = 1$ iff $|STUDENT \cap B| = 3$

Determiners denote functions – the basic idea

- `Exactly three` denotes that function $\mathbf{F}_2$, which when applied to a set A results in the function **F1**:

  $[\![\texttt{exactly three}]\!](A) = \mathbf{F1}$

- So, the expression `exactly three` denotes that function $\mathbf{F}_2$, which when applied first to a set A and then to a set B results in the truth value 1 iff:

  $[\mathbf{F}_2(A)](B) = 1 \text{ iff } |A \cap B| = 3$

Determiners denote functions – notation

Examples:

$[\![\texttt{exactly three}]\!] =_{def} \lambda A.\lambda B.|A \cap B| = 3$

$[\![\texttt{most}]\!] =_{def} \lambda A\lambda B.|A \cap B|/|A| \geq 0.75$

$[\![\texttt{no}]\!] =_{def} \lambda A.\lambda B.|A \cap B| = 0$

Determiners denote conservative functions

The truth of the expressions:

(1) `Exactlty three students smoke.`

(2) `Most students smoke.`

(3) `No student smokes.`

does not depend on the set of smokers which are not students, i.e. on the set consisting of the elements in the second set B which are not in the first set A.

Barwise and Cooper (1981), Keenan (1981) have hypothesised that all natural language determiners have this property:

$D(A)(B)$ iff $D(A)(A \cap B)$

Example of function excluded by conservativity

The hypothesis that all natural languages are conservative implies among other things that no language has a determiner denotation (expressed let's say by `rouf`), which when combined with a set A and then with the set B (e.g. fist with `student` and then with `smokes`) results in a true sentence if and only if the set of smokers which are not students is exactly four.

(4) `rouf student smokes.`
is true true iff $|SMOKE - STUDENT| = 4$

How can we explain this property of determiners?

Why are determiner denotations restricted to conservative functions?

Kamp and Reyle (1993, 317):

- determiners are binary relations between restrictor and scope set
- an independent principle guarantees that non-conservative determiners cannot be expressed.

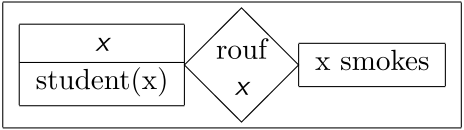
Conservativity and dynamic semantics

(5) `rouf student smokes.`

is true true iff $|SMOKE - STUDENT| = 4$

An embedding function f verifies the DRS

(6)



in M iff the relation $ROUF(A, B)$ iff $|B - A| = 4$ holds between the sets

- $A = \{a | g \supseteq_{U_{K_1}} f \cup \langle x, a \rangle \text{ verifies } student(x)\}$
- $B = \{a | h \supseteq_{U_{K_2}} g \text{ verifies } \text{ and } x\ smokes\}$

Conservativity and dynamic semantics

So, an embedding function f verifies the DRS (6) in model M iff the relation $ROUF(A, B)$ iff $|B - A| = 4$ holds between the sets

- $A = \{a | g \supseteq_{U_{K_1}} f \cup \langle x, a \rangle$ verifies $student(x)\}$
- $B = \{a | h \supseteq_{U_{K_2}} g$ verifies and x $smokes\}$

Crucial: the verifying assignments h are **extensions** of the verifying assignments g.

Therefore, every assignment h which satisfies the right-hand DRS also satisfies the left-hand DRS.

Therefore, only those entities are counted in B which are both smokers and students, and thus only the set of students and the set of students who smoke are available, and thus the non-conservative relation *ROUR* cannot be expressed.

An alternative idea

The denotation of a quantifier phrase does not refer to the scope set.

If the denotation of the determiner does not refer to the scope set, then the question why determiner denotations are restricted to conservative functions does not arise.

If this is correct, then conservativity is an artifact of the hypothesis that determiners are functions from restrictor and scope set into truth-values (or equivalently binary relations between restrictor and scope set)?

An asymmetry

In DRT the semantic contributions made by quantified NPs on the one hand and indefinite NPs and pronouns on the other hand are **different**.

However, the formal properties of quantified NPs, indefinite NPs and pronouns are **similar**.

- e.g. the expressions `most students`, `some students`, `they` can all occur in the environment _ `are happy`
- The expression `a student` can be the antecedent of the pronoun `he`, `most students` can be antecendent of pronoun `they`.

(7) `A student arrived. He was very late.`

(8) `Most students arrived. They were very late.`

Why should there be such an asymmetry?

Is this asymmetry between formal and semantic properties an artifact, too?

If the semantic contribution of quantified NPs is similar to the semantic contribution of indefinite NPs, i.e. the introduction of a variable and a condition on this variable, then we could argue that the reason for the formal similarity is to be found in the semantic similarity.

Semantic role assignment and determination of scope relations – different in nature?

Consider the sentence:

(9) `Five professors examined exactly twenty students.`

(10) `Five professors examined most students.`

Note that the observable formal properties of the sentence determine that the professors are the ones who examine and that the students are the ones who are examined **independently of the context of utterance**.

Semantic role assignment and determination of scope relations – different in nature?

However, the observable formal properties of the sentence do not determine whether:

- five professors are such that each of them examined twenty (possibly different) students
- twenty students are such that each was examined by five professors
- a totality of five professors stands in the examine relation to a totality of twenty students
- four professors examined together nineteen students, and another professor examined on his own the last remaining student
- etc.

Generalisation

Semantic role assignment is (generally) a context-independent aspect of the interpretation.

On the other hand, the precise relation between professors and students is (generally) varies with the context of utterance.

Constraint on empirical adequacy

When the semantic roles can be assigned independently of how the scope of the quantifiers is determined, then the theory should predict this.

If we aim for a theory which characterises the relation or interaction between context-independent and context-dependent content of utterances, then it is necessary to characterise the two types of content independently of one another.

However, . . .

According to the basic idea the assignment of the semantic role and the determination of the scope relations are intertwined, because semantic role assignment and scope relations are both determined at the same time – namely when the function denoted by the quantified NP is applied to its argument.

- Heim and Kratzer (1998): Semantics in generative grammar

  Quantified NPs are always of type $\langle\langle e, t\rangle, t\rangle$, and scope is determined at the point of combination

- Keenan (1987): Semantic case theory

  Quantified NPs have a generalised type: they take an n-ary relation and return an $n-1$-ary relation. Again, scope is determined when the quantified NP denotation combines with its argument.

Cooper storage

Splits the semantic combination of quantified NP denotations and predicate into two steps:

1. all quantified NP denotations are assigned a semantic role (by means of co-indexing a semantic role of a predicate with a quantified NP denotation) and then put on store
2. quantified NP denotations are combined with the predicate by means of functional application; scope relations between quantified NP denotations depends on the order in which the denotations in the store merge with the predicate.

A promisory note

If quantified NPs introduce variables and conditions on the instantiating entity, then the semantic role could be assigned to the variable independently of how (i) ascriptions of properties to restricted sets or (ii) relations between restricted sets are evaluated.

Intermediate summary

- If quantified NPs do not refer to the scope set, then the question why the truth of quantified sentences never depends on the set of entities in the scope which are not in the restrictor does not arise.
- If qantified and indefinite NPs make similar semantic contributions then it is not unexpected that they should have similar formal properties, and we are one step closer to a uniform semantic analysis of NPs.
- If quantified NPs introduce a set variable, then this set variable can be assigned a semantic role independently of how the scope relations are determined.

Quantified NPs partition restrictor set

`most students`

Variable: X (a set of entities)

Restrictor: ST

Condition: $|X|/|ST| \geq 0.75$

$X \mid ST$
$X \subseteq ST$ $\lvert X\rvert/\lvert ST\rvert \geq 0.75$

Quantified NPs partition restrictor set

`exactly three students`

Variable: X

Restrictor: ST

Condition: $|X| = 3$

$X \mid ST$
$X \subseteq ST$ $\lvert X \rvert = 3$

Quantified NPs partition restrictor set

`no student`

Variable: X

Restrictor: ST

Condition: $|X| = 0$

$X \| ST$
$X \subseteq ST$ $\|X\| = 0$

Quantified NPs partition restrictor set

`(all of) John's three articles`

Variable: X

Restrictor: $ART \cap \{x|R(j,x)\}$

Condition: $X = ART \cap \{x|R(j,x)\} \wedge |X| = 3$

$X \mid ST$
$X = ART \cap \{x\|R(j,x)\}$ $\|X\| = 3$

Some observations

Some conditions refer to the restrictor set:

- `all of John's three articles`
- `most students`

Some conditions do not refer to the restrictor set (intersective quantifiers):

- `exactly three students`
- `no student`

Only relations between a restrictor set and a subset of this restrictor set can be specified.

Therefore, the non-conservative function $[\![\texttt{rouf}]\!]$ cannot be expressed, since this relation cannot be specified as holding between a restrictor set and a subset X of the restrictor set.

Semantic role assignment

Predicates are unsaturated semantic entities, but unlike in the functional view, I assume that they do not specify the order in which the dependencies of the unsaturated entity are to be satisfied (the order in which meanings can combine is NOT part of the meanings themselves).

There are different types of dependencies (dependencies = semantic roles).

Semantic operations access semantic roles by means of their type, not by means of the order in which the semantic roles must be assigned.

Combining the object

$X\|ST$
$\|X\|/\|ST\| \geq 0.75$

$\bigoplus_2$

$x\ y$
$INSULT(1:x, 2:y)$

$=$

$x\ X\|ST$
$\|X\|/\|ST\| \geq 0.75$ $INSULT(1:x, X\|ST)$

Combining the subject

$X \mid ST$
$\lvert X \rvert / \lvert ST \rvert \geq 0.75$

$\oplus_1$

$x \; y$
$INSULT(1 : x, 2 : y)$

$=$

$X \mid ST \;\; y$
$\lvert X \rvert / \lvert ST \rvert \geq 0.75$ $INSULT(X \mid ST, 2 : y)$

Evaluating the ascription of properties to restricted sets

`Most students sing.`

$X\|ST$
$\|X\|/\|ST\| \geq 0.75$ $SING(X\|ST)$

X
$\|X\|/\|ST\| \geq 0.75$ $X = SING \cap ST$

iff $\exists X.|X|/|ST| \geq 0.75 \wedge X = ST \cap SING$

Evaluating relations between individuals and restricted sets

`John insulted most students.`

$x\ Y\|ST$
$John(x)$ $\|Y\|/\|ST\| \geq 0.75$ $INSULT(x, Y\|ST)$

$x\ Y$
$John(x)$ $\|Y\|/\|ST\| \geq 0.75$ $Y = ST \cap \{y\|INSULT(x, y)\}$

iff

$\exists x \exists Y . John(x) \wedge |Y|/|ST| \geq 0.75 \wedge Y = ST \cap \{y|INSULT(x, y)\}$

Evaluating relations between restricted sets – first (logical) possibility

`All professors insulted ten students.`

$X \vert PROF\ \ Y \vert ST$
$X = PROF$
$\vert Y \vert = 10$
$INSULT(X \vert PROF, Y \vert ST)$

$X\ \ Y \vert ST$
$X = PROF$
$\vert Y \vert = 10$
$X = PROF \cap \{x \vert INSULT(x, Y \vert ST)\}$

First possibility continued

`All professors insulted ten students.`

$$\begin{array}{|c|}\hline X \;\; Y|ST \\ \hline X = PROF \\ |Y| = 10 \\ X = PROF \cap \{x|INSULT(x, Y|ST)\} \\ \hline \end{array}$$

$$\text{iff } \exists X.X = PROF \wedge X = PROF \cap \{x|\begin{array}{|c|}\hline Y|ST \\ \hline |Y| = 10 \\ INSULT(x, Y|ST) \\ \hline \end{array}\}$$

$$\text{iff } \exists X.X = PROF \wedge X = PROF \cap$$
$$\{x|\exists Y.|Y| = 10 \wedge Y = ST \cap \{y|INSULT(x, y)\}\}$$

Second (logical) possibility

`All professors insulted ten students.`

$$\begin{array}{|c|}\hline X|PROF\ \ Y|ST \\ \hline X = PROF \\ |Y| = 10 \\ INSULT(X|PROF, Y|ST) \\ \hline \end{array}$$

$$\begin{array}{|c|}\hline X|PROF\ \ Y \\ \hline X = PROF \\ |Y| = 10 \\ Y = ST \cap \{y|INSULT(X|PROF, y)\} \\ \hline \end{array}$$

Second possibility continued

`All professors insulted ten students.`

$$\begin{array}{|c|}\hline X|PROF\ \ Y \\ \hline X = PROF \\ |Y| = 10 \\ Y = ST \cap \{y|INSULT(X|PROF, y)\} \\ \hline \end{array}$$

$$\text{iff } \exists Y.|Y| = 10 \wedge Y = ST \cap \{y| \begin{array}{|c|}\hline X|PROF \\ \hline X = PROF \\ INSULT(X|PROF, y) \\ \hline \end{array} \}$$

$$\text{iff } \exists Y.|Y| = 10 \wedge Y = ST \cap \{y|\exists X.X = PROF \wedge X = PROF \cap \{x|INSULT(x, y)\}\}$$

Conclusion

- argued against the analysis of determiners as make reference both to restrictor and to scope set
- argued for an analysis according to which determiners partition restrictor sets by introducing a set variable X, a restrictor and a condition on the set X
- this provides an alternative explanation for why functions like $[\![\texttt{rouf}]\!]$ cannot occur
- like in the Cooper Storage analysis, the assignment of the semantic role is decoupled from the resolution of the scope relations
- Bonus: analysis of E-type pronouns is a consequence of the theory