Partial semantics for iterated *if*-clauses

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The following two sentences seem to be equivalent (both are trivial) (Edgington 1995):

(1) a. If it rains or snows tomorrow, then if it doesn’t rain tomorrow, it will snow.
   b. If it rains or snows tomorrow and it doesn’t rain, it will snow.

Let $\rightarrow$ be the indicative conditional connective. Gibbard (1981) famously proved that if the following principles are assumed, it follows from the equivalence of $p \rightarrow (q \rightarrow r)$ and $(p \land q) \rightarrow r$ that $\rightarrow$ must be material implication:

(i) $\phi \rightarrow (\psi \rightarrow \chi) \equiv (\phi \land \psi) \rightarrow \chi$
(ii) $\phi \rightarrow \psi \models \phi \supset \psi$
(iii) If $\models \psi$, then $\models \phi \rightarrow \psi$

But it is well-known that if we analyze indicative conditionals in terms of material implication, we run into paradoxes (Bennett 2003). So it seems that we can either have a semantics for indicatives which gives us the equivalence of (1a) and (1b), or a non-paradoxical semantics for indicatives, but not both.

Kratzer (1991) argued that Gibbard’s proof holds only if one takes indicative conditionals to be two-place operators. In her framework, however, the logical forms for natural language indicatives have a very different structure. She treats *if*-clauses as restrictions on the domain of a quantificational operator. If there is no explicit operator present, we must posit one; usually an epistemic necessity modal. Thus, (2a) and (2b) are treated as equivalent, which seems right:

(2) a. If Harry is not in the library, he is in the common room.
   b. If Harry is not in the library, he must be in the common room.

Both sentences receive the following logical form:

(3) (must: Harry is not in the library)(he is in the common room)

This is true iff Harry is in the common room in all worlds that are compatible with the available evidence and in which Harry is not in the library.

What about stacked *if*-clauses? Kratzer proposes that such *if*-clauses successively restrict the domain of one and the same quantifier, so that the logical form of both (1a) and (1b) is:

(4) (must: it rains or snows and it doesn’t rain)(it snows)

But it is obvious that to derive (4) from (1a), we must drastically rearrange various parts of the sentence. In particular, the presence of *then* is mysterious. For this reason,
Schein (2003) and Schlenker (2004) have proposed to treat *if*-clauses as plural definite descriptions (of events and worlds respectively). This leads to a syntactically more natural analysis, yet makes slightly different predictions than Kratzer. For instance, it is now expected that modals can also collectively quantify over the events or worlds supplied by the *if*-clause. It isn’t clear that this ever happens.

There is an alternative, which, other than Kratzer’s analysis and Schlenker’s proposal, does assume conditional connectives in the logical form of indicatives. Following Belnap (1970), we could assign partial truth conditions to →:

\[
\llbracket \phi \rightarrow \psi \rrbracket^w = \llbracket \psi \rrbracket^w \text{ if } \llbracket \phi \rrbracket^w = 1; \text{ otherwise } \llbracket \phi \rightarrow \psi \rrbracket^w \text{ is undefined.}
\]

If we now assume that quantificational operators are restricted to worlds in which the embedded proposition has a truth value, embedding → under a modal has the effect of restricting that modal’s domain with the *if*-clause. Thus, (2b) may be analyzed as follows:

\[
\text{(must: })(\text{Harry is not in the library } \rightarrow \text{ he is in the common room})
\]

This is true if in all worlds in which the conditional is defined, i.e. in which Harry isn’t in the library, he is in the common room. We thus make the same predictions as Kratzer’s theory.

In addition, this semantics elegantly handles iterated *if*-clauses. (1a) can be analyzed as follows:

\[
\text{(it rains or snows) } \rightarrow \text{ (it doesn’t rain } \rightarrow \text{ it snows)}
\]

If (7) has a truth value, i.e. if it rains or snows, then it snows if the embedded conditional has a truth value, i.e. if it doesn’t rain. That is, it snows if it rains or snows but doesn’t rain. So (1a) comes out equivalent to (1b). Note that on this theory, there is no mismatch between syntax and semantics, as this representation follows the surface form of (1a) in a straightforward manner.

Of course, on this semantics, neither of (1a) and (1b) comes out as trivial, because they may be undefined (if the antecedent is not true). We can, however, explain away the intuition that these sentences are tautological, by pointing out that they are *maximally assertable*. Following McDermott (1996), a conditional’s assertability is the probability that it is true given that it has a truth value. Clearly, (7) is maximally assertable: if it has a truth value, it is most definitely true. We just thought that the sentence was trivial, because we tend to confuse maximal assertability with necessary truth.

Summing up, there is another way to avoid the conclusion that if we want to have the equivalence between (1a) and (1b), material implication is the only candidate for indicative conditionals. We can assign partial truth conditions to indicatives. This suggests that Gibbard’s proof only holds in a classical, two-valued system. Note that the culprit is his assumption (iii). Given Belnap’s semantics, (iii) is simply not true. If all models that make \( \phi \) true in all worlds make \( \psi \) true in all worlds, we cannot conclude that all models make \( \phi \rightarrow \psi \) true in all worlds, for such models may contain \( \neg \phi \)-worlds and in these worlds, \( \phi \rightarrow \psi \) has no truth value.

**References**


